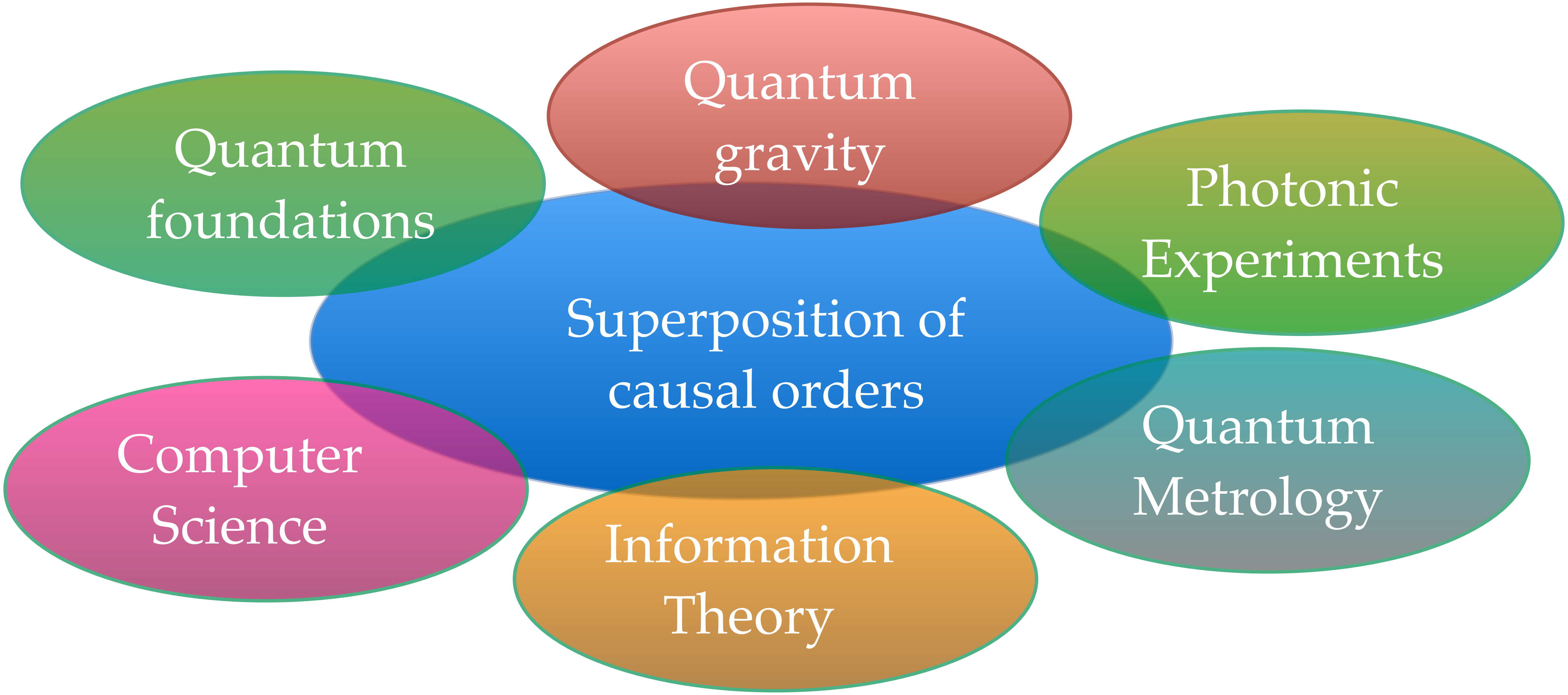
QUANTUM SUPERPOSITION OF CAUSAL ORDERS

Giulio Chiribella

QICI, The University of Hong Kong

PhD Course, University of Palermo, 28 Nov 2022

CAUSAL STRUCTURE IN QUANTUM PHYSICS



FROM GRAVITY TO QUANTUM INFORMATION

General relativity is
a deterministic theory with non-fixed causal structure.
Quantum theory is
a probabilistic theory with fixed causal structure.
In this paper we build a framework
for probabilistic theories with non-fixed causal structure.
This combines the radical elements
of general relativity and quantum theory.

Journal of Physics A: Mathematical and Theoretical

Towards quantum gravity: a framework for probabilistic theories with non-fixed causal structure

Lucien Hardy

Published 7 March 2007 • 2007 IOP Publishing Ltd

Journal of Physics A: Mathematical and Theoretical, Volume 40, Number 12

Lucien Hardy, Perimeter Institute

SpringerLink

Quantum Reality, Relativistic Causality, and Closing the Epistemic Circle pp 379-401 | Cite as

Quantum Gravity Computers: On the Theory of Computation with Indefinite Causal Structure

Authors | Authors and affiliations

Lucien Hardy

SUPERPOSITION OF CAUSAL STRUCTURES

Beyond Quantum Computers

G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron

The manuscript poses and addresses a new very fundamental issue in Quantum Computer Science, which is going to have a radical impact on the way we currently conceive quantum computation. We show that there exists a new kind of "higher-order" quantum computation, potentially much more powerful than the usual quantum processing, which is feasible, but cannot be realized by a usual quantum circuit. In order to implement this new kind of computations one needs to change the rules of quantum circuits, also considering circuits with the geometry of the connections that can be itself in a quantum superposition. The new kind of computation poses also fundamental problems for unexplored aspects of quantum mechanics in a non-fixed causal framework, which go far beyond computer-science problems, and may be of relevance in quantum gravity.

arXiv:0912.0195

Quantum computations without definite causal structure

Giulio Chiribella, Giacomo Mauro D'Ariano, Paolo Perinotti, and Benoit Valiron
Phys. Rev. A **88**, 022318 – Published 14 August 2013

We show that quantum theory allows for transformations of black boxes that cannot be realized by inserting the input black boxes within a circuit in a predefined causal order. The simplest example of such a transformation is the *classical switch of black boxes*, where two input black boxes are arranged in two different orders conditionally on the value of a classical bit. The quantum version of this transformation—the *quantum switch*—produces an output circuit where the order of the connections is controlled by a quantum bit, which becomes entangled with the circuit structure. Simulating these transformations in a circuit with fixed causal structure requires either postselection or an extra query to the input black boxes.

Phys. Rev. A 88, 022318 (2013)

THE QUANTUM SWITCH

An hypothetical machine that combines two black boxes in a coherent superposition of alternative orders.

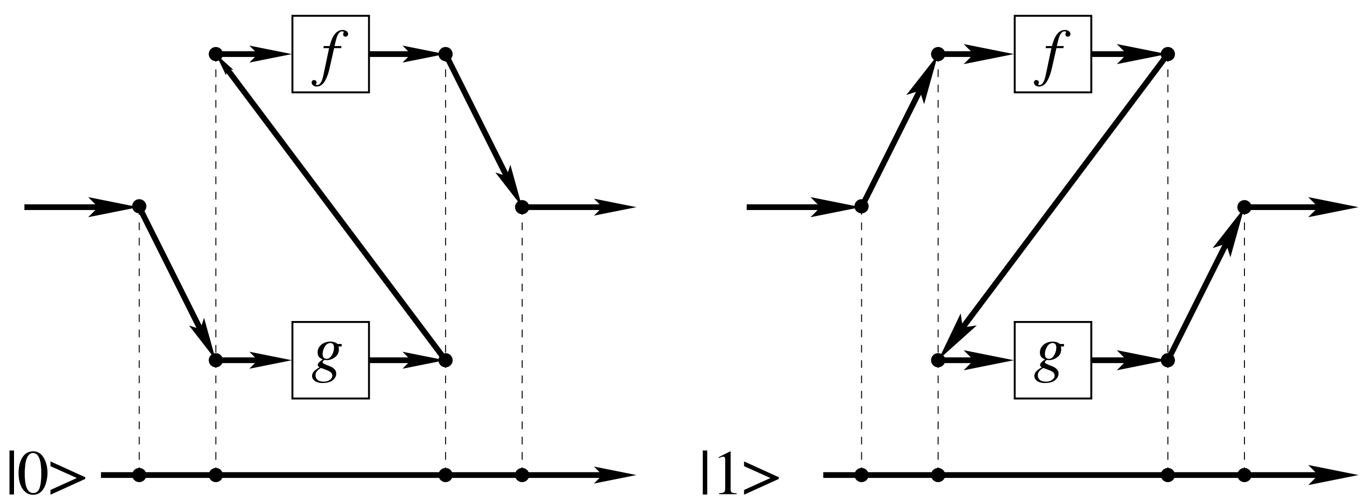


FIG. 1: Quantum machine with classical control over movable wires.

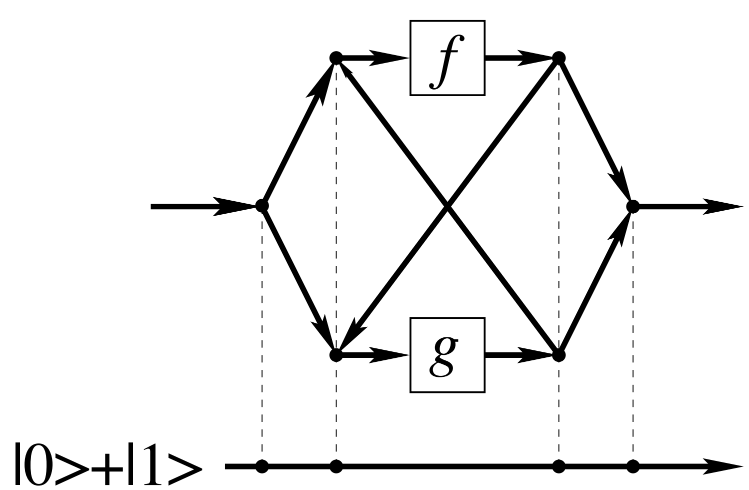


FIG. 2: Quantum machine with quantum control over movable wires.

figures from arXiv:0912.0195

INFORMATION-THEORETIC ADVANTAGES

Open Access | Published: 02 October 2012

Quantum correlations with no causal order

Ognyan Oreshkov, Fabio Costa & Časlav Brukner

Nature Communications 3, Article number: 1092 (2012) | Cite this article

Rapid Communication

Perfect discrimination of no-signalling channels via quantum superposition of causal structures

Giulio Chiribella
Phys. Rev. A **86**, 040301(R) – Published 10 October 2012

Editors' Suggestion

Computational Advantage from Quantum-Controlled Ordering of Gates

Mateus Araújo, Fabio Costa, and Časlav Brukner
Phys. Rev. Lett. **113**, 250402 – Published 18 December 2014

Enhanced Communication with the Assistance of Indefinite Causal Order

Daniel Ebler, Sina Salek, and Giulio Chiribella
Phys. Rev. Lett. **120**, 120502 – Published 22 March 2018

Quantum Metrology with Indefinite Causal Order

Xiaobin Zhao, Yuxiang Yang, and Giulio Chiribella
Phys. Rev. Lett. **124**, 190503 – Published 14 May 2020

Quantum Refrigeration with Indefinite Causal Order

David Felce and Vlatko Vedral
Phys. Rev. Lett. **125**, 070603 – Published 11 August 2020

EXPERIMENTS

Experimental superposition of orders of quantum gates

Lorenzo M. Procopio, Amir Moqanaki, Mateus Araújo, Fabio Costa, Irati Alonso Calafell, Emma G. Dowd, Deny R. Hamel, Lee A. Rozema, Časlav Brukner & Philip Walther

Nature Communications **6**, Article number: 7913 (2015) | Cite this article

Experimental verification of an indefinite causal order

Giulia Rubino[1,*], Lee A. Rozema[1], Adrien Feix[1,2], Mateus Araújo[1,2], Jonas M. Zeuner[1], Lorenzo ...
+ See all authors and affiliations

Science Advances 24 Mar 2017:
Vol. 3, no. 3, e1602589
DOI: 10.1126/sciadv.1602589

Editors' Suggestion

Indefinite Causal Order in a Quantum Switch

K. Goswami, C. Giarmatzi, M. Kewming, F. Costa, C. Branciard, J. Romero, and A. G. White
Phys. Rev. Lett. **121**, 090503 – Published 31 August 2018

+ recent review

Experiments on quantum causality

AVS Quantum Sci. **2**, 037101 (2020); https://doi.org/10.1116/5.0010747

K. Goswami[a)] and J. Romero[b)]

Experimental Quantum Switching for Exponentially Superior Quantum Communication Complexity

Kejin Wei, Nora Tischler, Si-Ran Zhao, Yu-Huai Li, Juan Miguel Arrazola, Yang Liu, Weijun Zhang, Hao Li, Lixing You, Zhen Wang, Yu-Ao Chen, Barry C. Sanders, Qiang Zhang, Geoff J. Pryde, Feihu Xu, and Jian-Wei Pan
Phys. Rev. Lett. **122**, 120504 – Published 28 March 2019

Experimental Transmission of Quantum Information Using a Superposition of Causal Orders

Yu Guo, Xiao-Min Hu, Zhi-Bo Hou, Huan Cao, Jin-Ming Cui, Bi-Heng Liu, Yun-Feng Huang, Chuan-Feng Li, Guang-Can Guo, and Giulio Chiribella
Phys. Rev. Lett. **124**, 030502 – Published 24 January 2020

PLAN OF THE TALK

- Foundations: quantum supermaps and the quantum SWITCH

- Applications: query complexity, quantum communication, quantum metrology

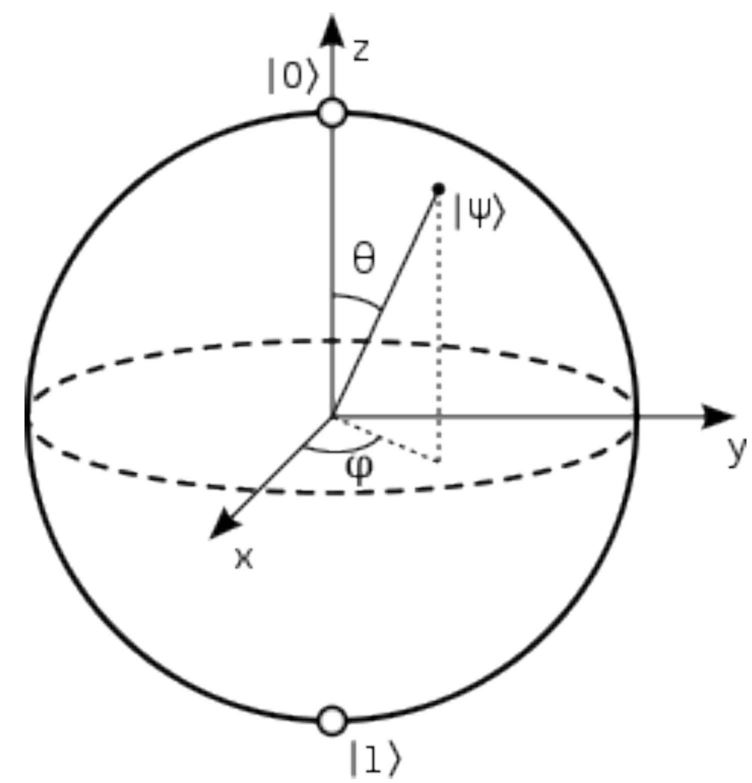
THEORETICAL FRAMEWORK: QUANTUM SUPERMAPS

To see a World in a Grain of Sand
And a Heaven in a Wild Flower
Hold Infinity in the palm of your hand
And Eternity in an hour.

William Blake, ca. 1803

FORGET EVERYTHING, EXCEPT QUANTUM STATES

Promise: there exist quantum systems.



Quantum system ⟶ Hilbert space $\mathscr{H} = \mathbb{C}^d$

Quantum states = density matrices

$$\rho \in L(\mathscr{H})\,, \quad \boxed{\langle\psi|\rho|\psi\rangle \geq 0 \quad \forall\, |\psi\rangle \in \mathscr{H}\,,} \quad \mathrm{Tr}[\rho] = 1$$

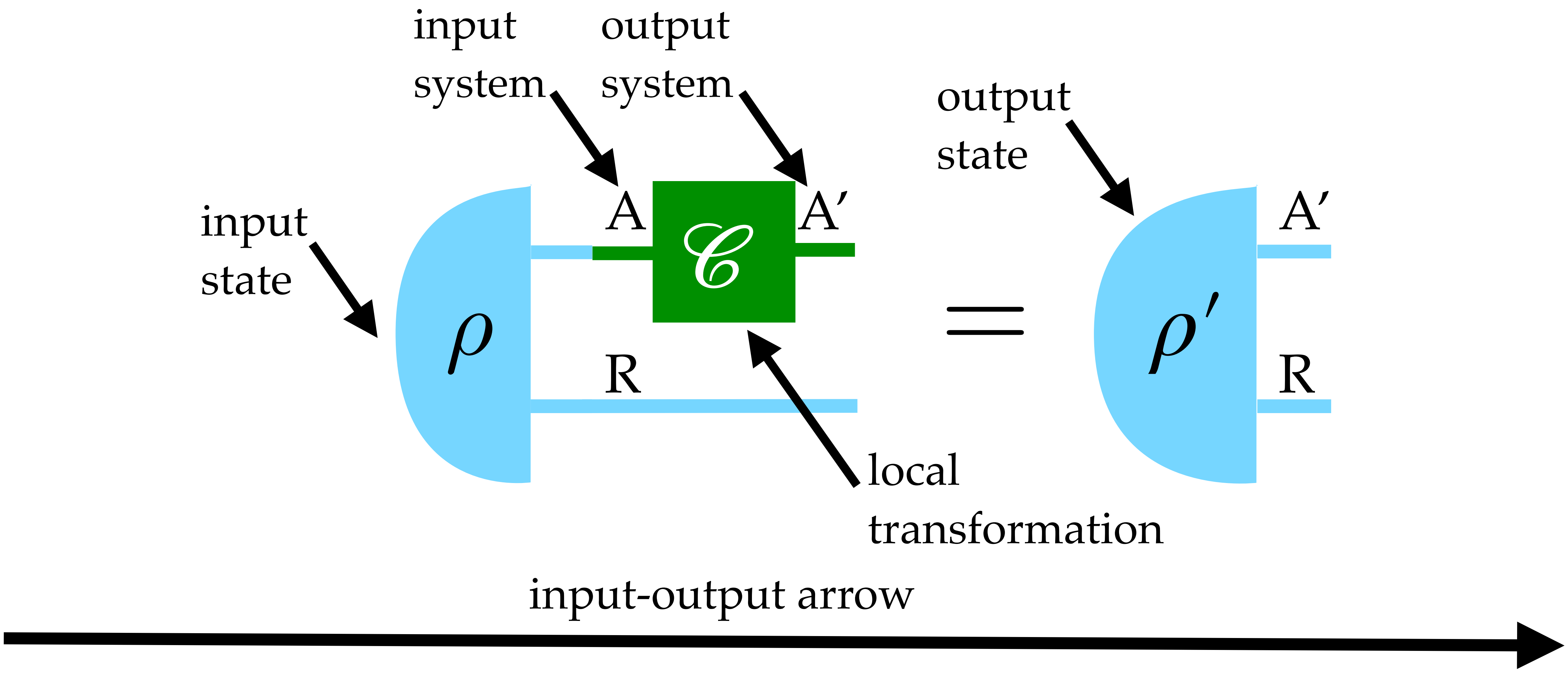
$$\rho \geq 0$$

Question:

What is the most general way
to transform
quantum states into quantum states?

ADMISSIBLE MAPS

Admissible map: must be **linear*** and **send states into states**, even when acting locally on one part of a composite system



*why linear?
see **GC, D'Ariano, and Perinotti,** ***Quantum from Principles*****, in** ***Quantum Theory: Informational Foundations and Foils*****, Springer 2016, arXiv:1506.00398, p. 11**

EXAMPLE AND NON-EXAMPLE

- Example: unitary map $\mathscr{U}(\rho) := U\rho U^\dagger, \quad U^\dagger U = UU^\dagger = I$

- Non-example: transpose map $\Theta(\rho) := \rho^T \quad \forall \rho$

Apply it to $|\Phi^+\rangle := \sum_k |k\rangle \otimes |k\rangle / \sqrt{d}$,

get non-positive matrix.

CHARACTERIZATION OF THE ADMISSIBLE MAPS

Every admissible map has a *Kraus representation*

$$\mathscr{C}(\rho) = \boxed{\sum_i C_i \, \rho \, C_i^\dagger} \quad \text{with} \quad \boxed{\sum_i C_i^\dagger C_i = I}$$

completely positive

trace-preserving

Admissible maps = completely positive, trace-preserving maps

=: quantum channels

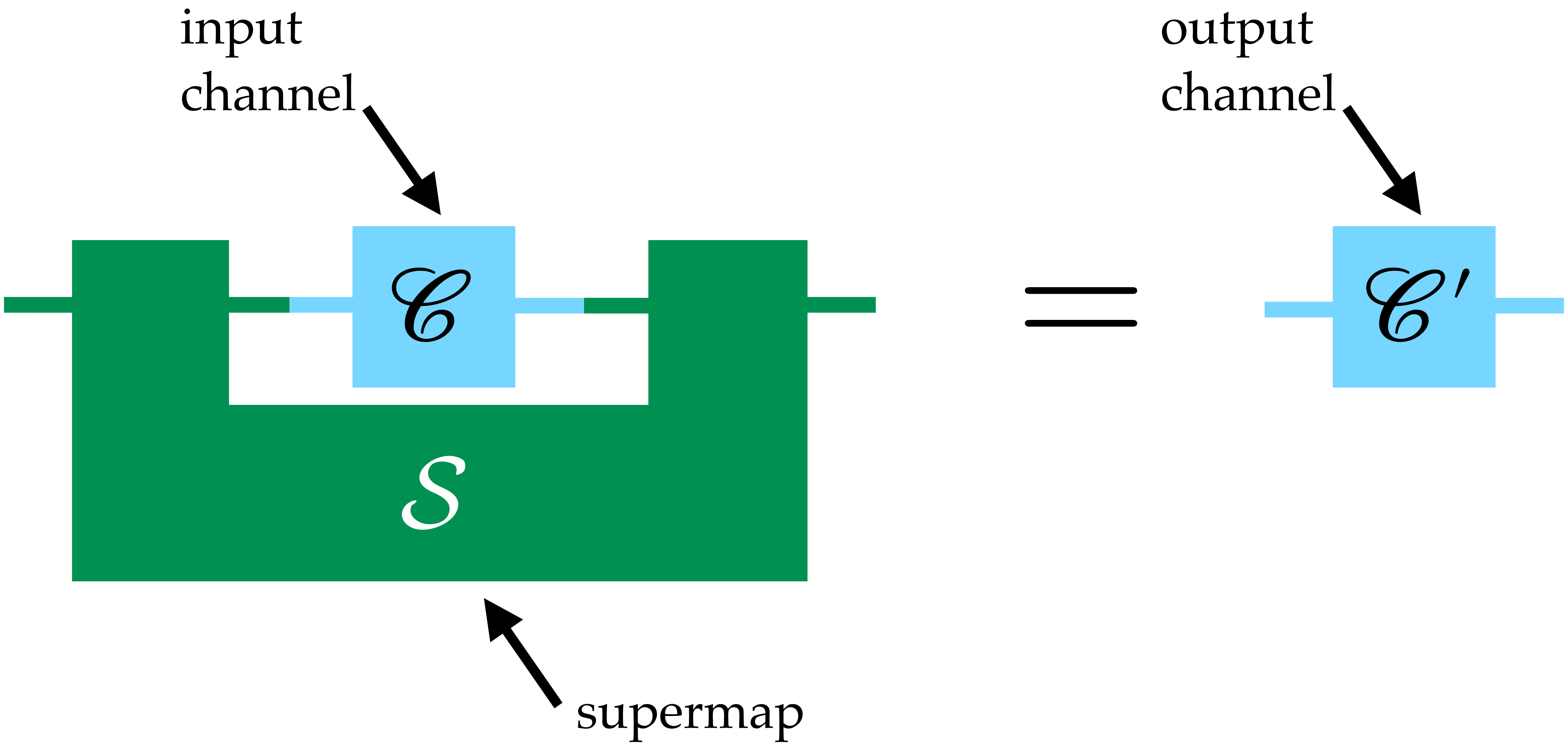
cf. Heinosaari and Ziman, Cambridge University Press (2011)

Next Level:

What is the most general way
to transform
quantum channels into quantum channels?

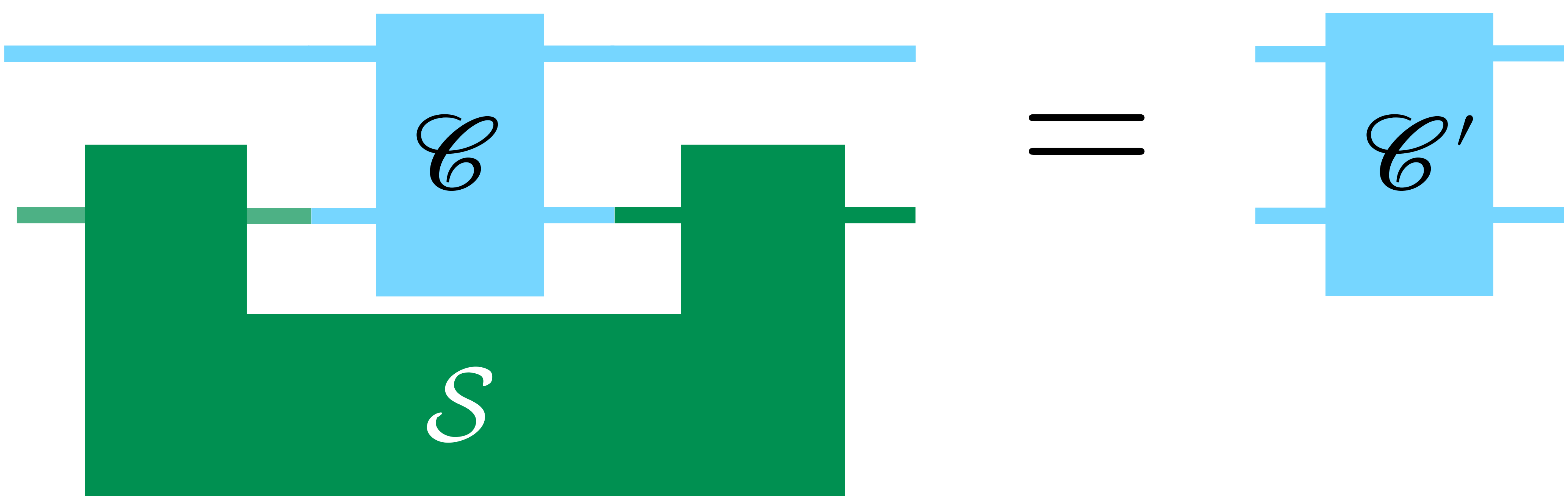
SUPERMAPS

Supermaps = transformations of quantum channels



ADMISSIBLE SUPERMAPS

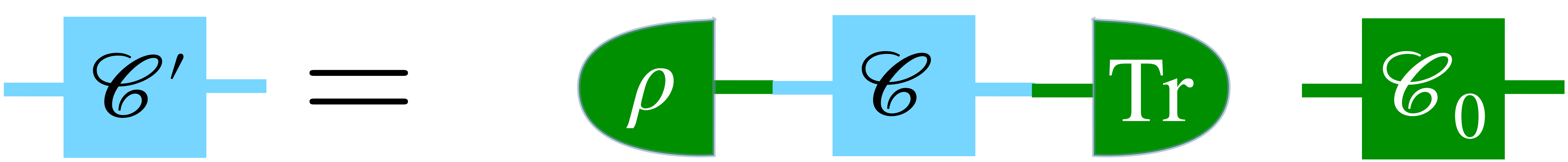
Admissible supermap: must be **linear***
and **send channels into channels**,
even when acting locally on one part of
a bipartite channel



EXAMPLES

- Encoding-decoding $\mathscr{S}(\mathscr{C}) := \mathscr{D} \circ \mathscr{C} \circ \mathscr{E}$,
  with $\mathscr{E}$ and $\mathscr{D}$ fixed quantum channels

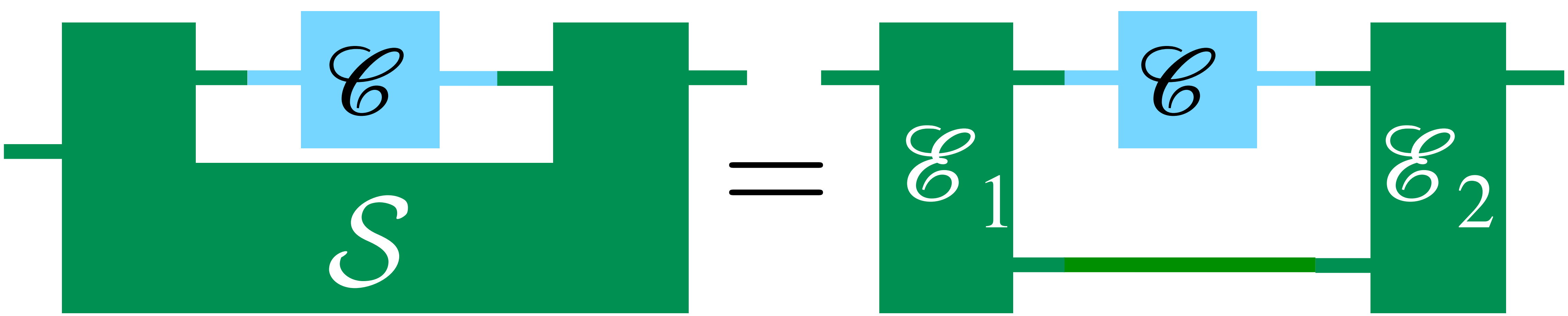
- Replacement $\mathscr{S}(\mathscr{C}) := \mathrm{Tr}[\mathscr{C}(\rho)]\,\mathscr{C}_0$,
  with ρ fixed state and $\mathscr{C}_0$ fixed channel



CHARACTERIZATION
OF THE ADMISSIBLE SUPERMAPS

Theorem

Every admissible supermap can be realized by a network of channels with memory:

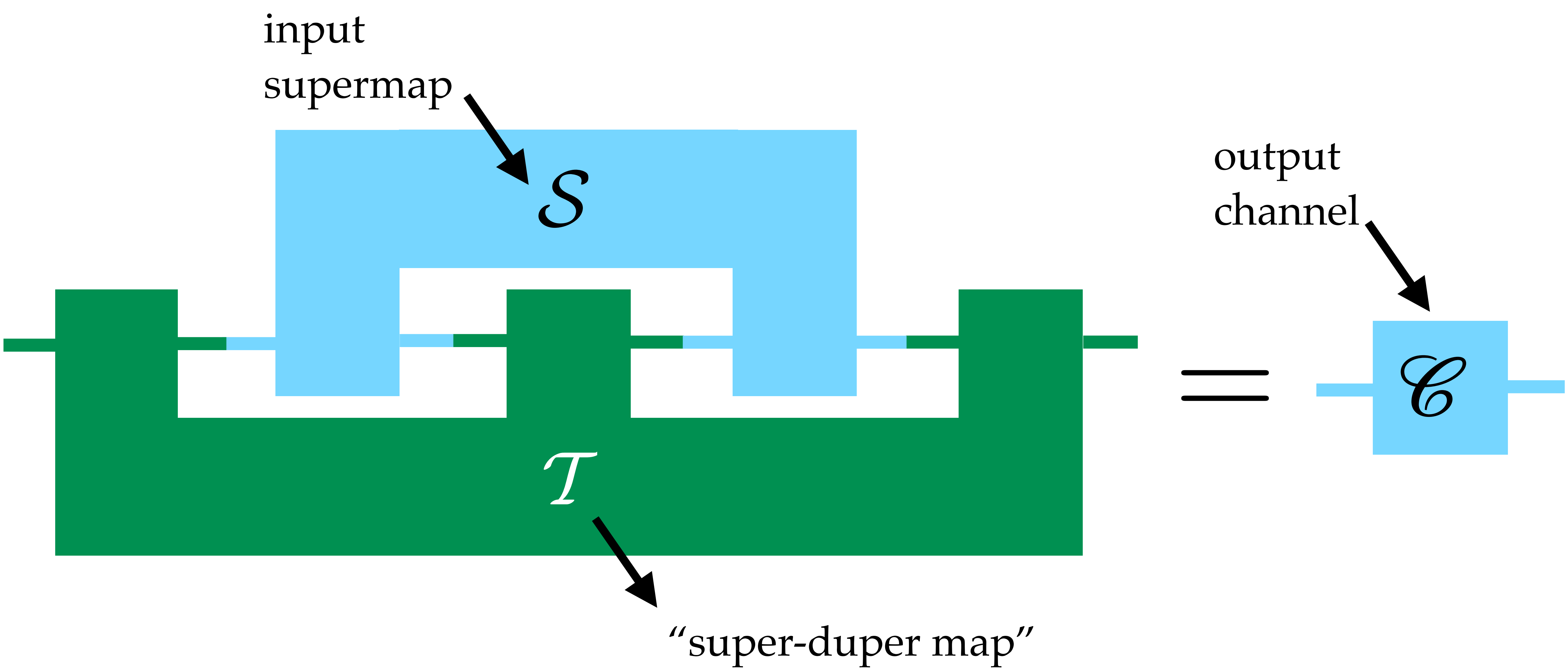


Chiribella, D'Ariano, and Perinotti, EPL 83, 30004 (2008)

Next Next Level:

What is the most general way
to transform
admissible supermaps into admissible channels?

HIGHER-ORDER SUPERMAPS

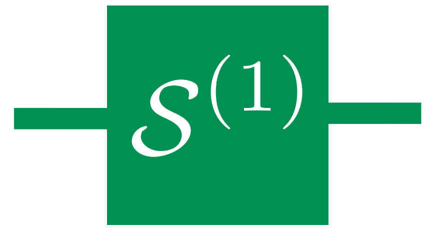


Admissible “super-duper map”:

must be **linear*** and **send admissible supermaps into channels**, even when acting locally on one part of a bipartite supermap

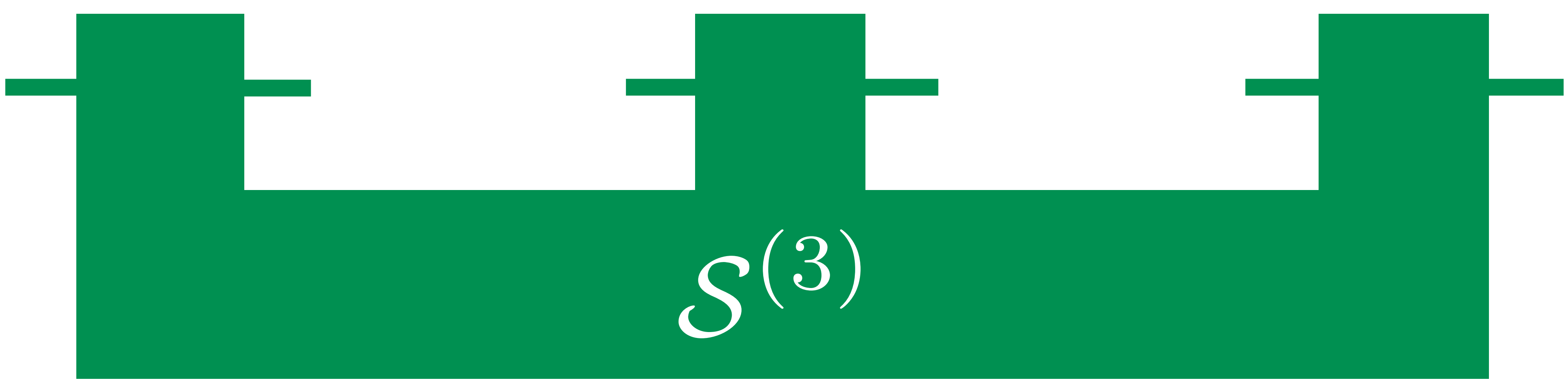
ADMISSIBLE N-MAPS

N=1 quantum channel



N=2

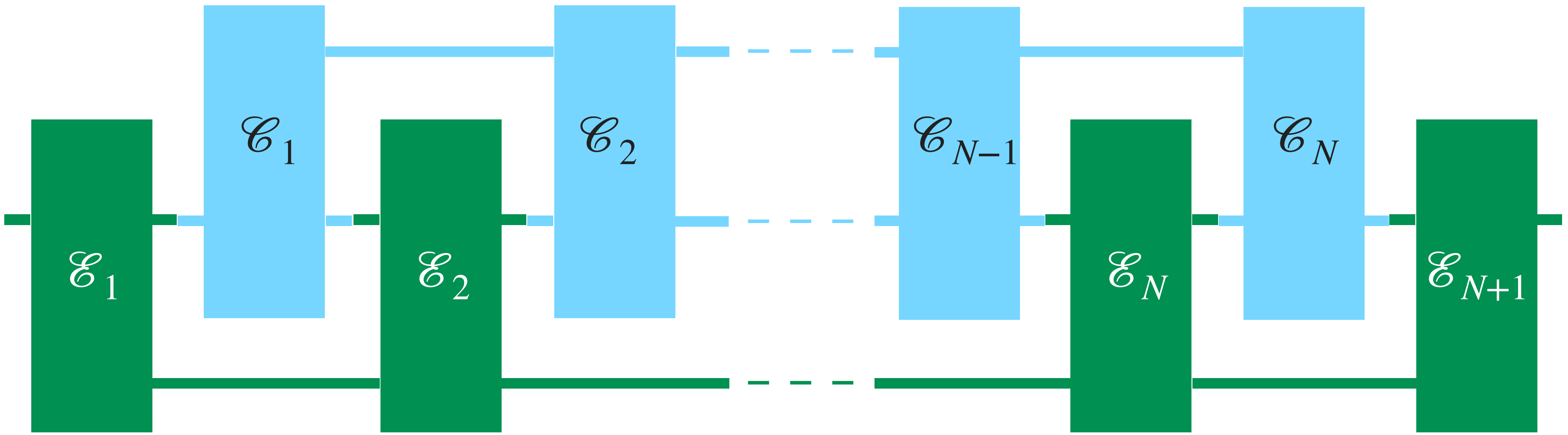
N=3



REALIZATION OF ADMISSIBLE N-MAPS

Theorem

Any admissible N-map can be realized by a sequential network of quantum channels with memory:



Chiribella, D'Ariano, and Perinotti, Phys. Rev. A 80, 022339 (2009)

Getting to the weird levels:

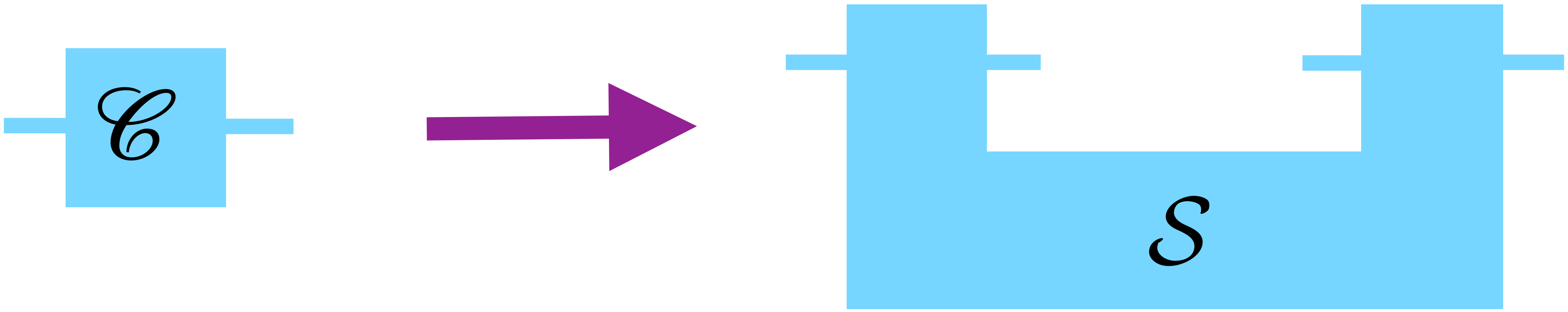
What is the most general way

to transform

admissible supermaps into admissible supermaps?

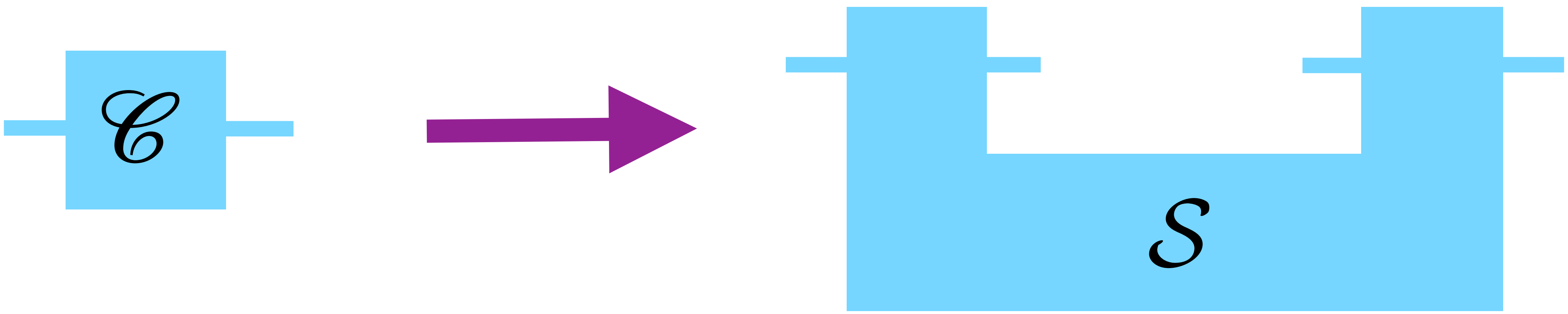
THE EASIEST EXAMPLE

Question: what is the most general way to transform a quantum channel into a supermap?

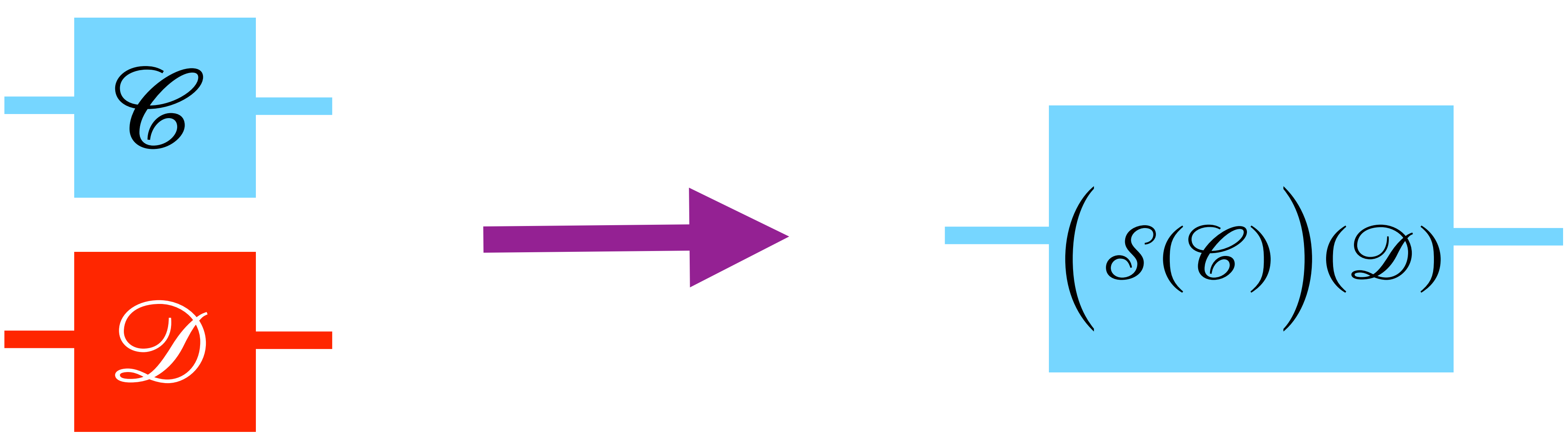


EQUIVALENT FORMULATION

Transforming a channel into a supermap



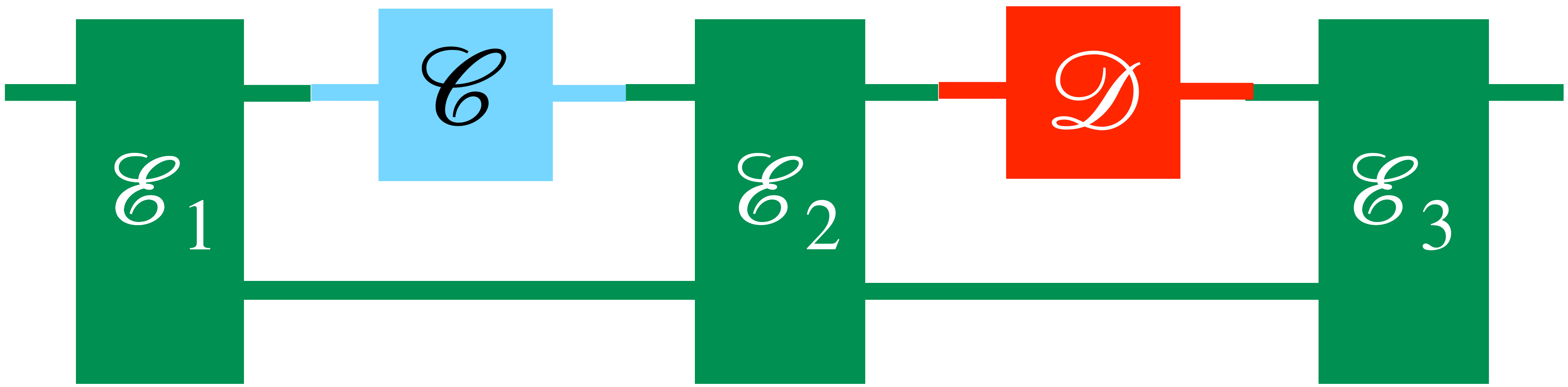
is equivalent to
transforming a *pair* of channels into a channel



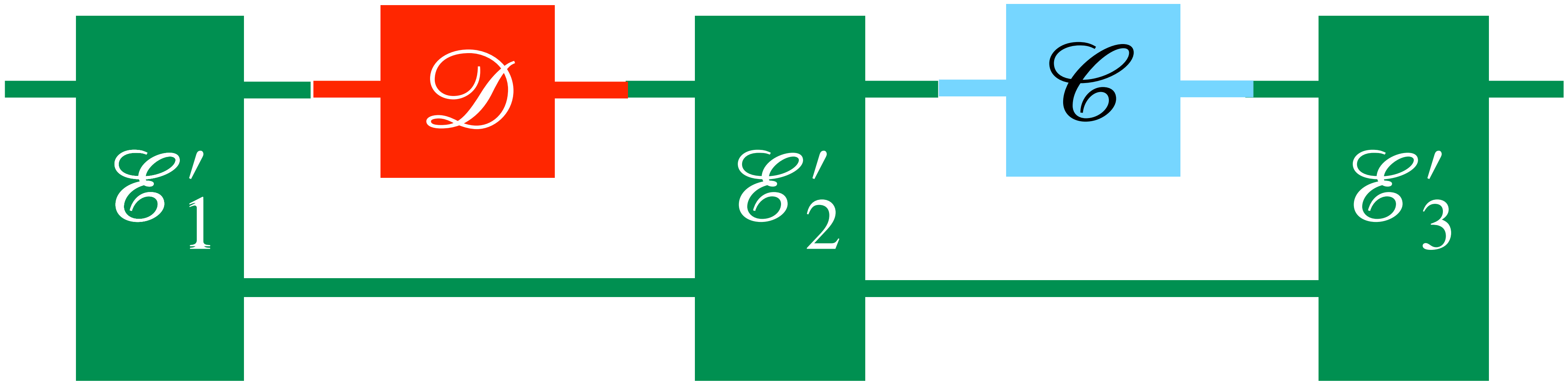
TWO COMPLEMENTARY ORDERS

There are two alternative causal networks.

- First realization: place $\mathscr{C}$ before $\mathscr{D}$

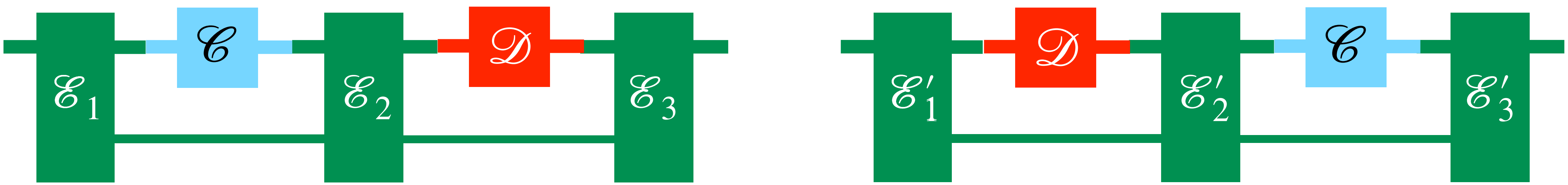


- Second realization: place $\mathscr{D}$ before $\mathscr{C}$



MIXTURE VS SUPERPOSITION OF CAUSAL STRUCTURES

Two complementary choices of causal networks:



We could choose randomly between these two supermaps.

But we can also choose coherently, depending on the state of a control qubit. This gives us a coherent superposition of causal structures.

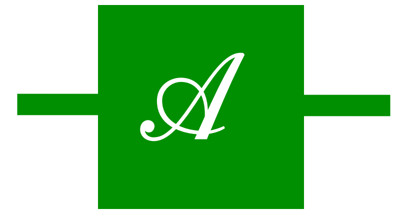
THE

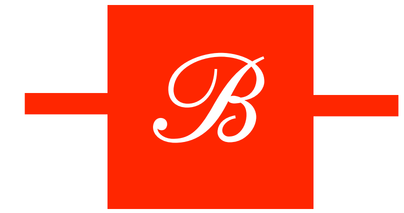
QUANTUM

SWITCH

THE (SIMPLIFIED) QUANTUM SWITCH

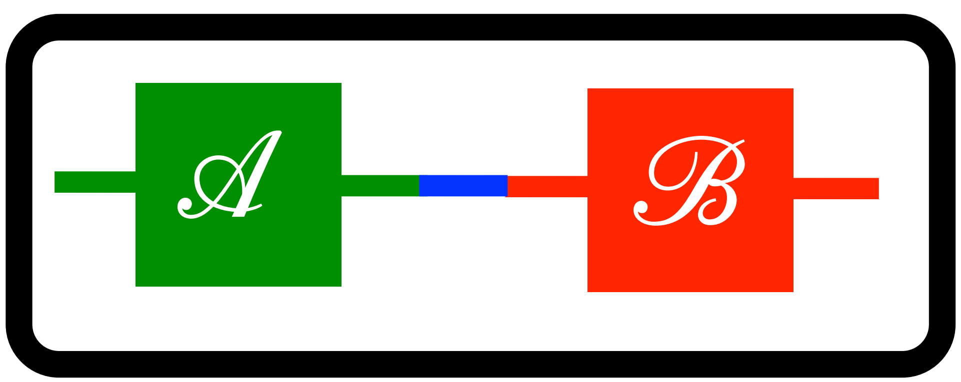
The *(simplified)* quantum SWITCH is the supermap that

takes as input the two processes 

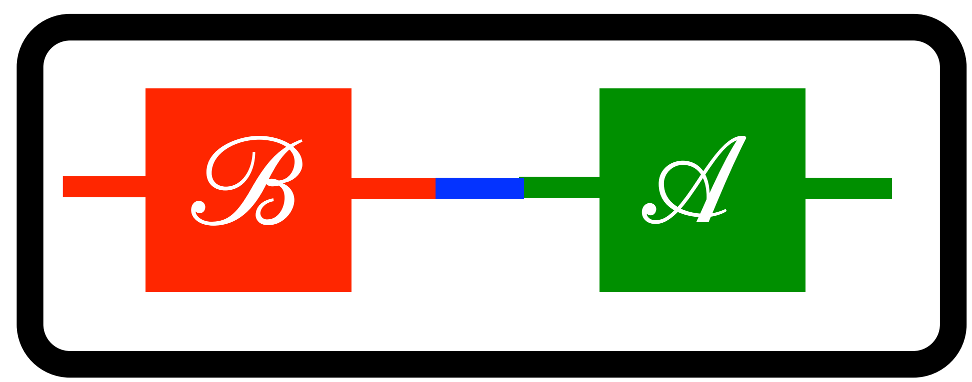
and 

with equal inputs/outputs

and *connects* them in a coherent superposition of the two configurations



and



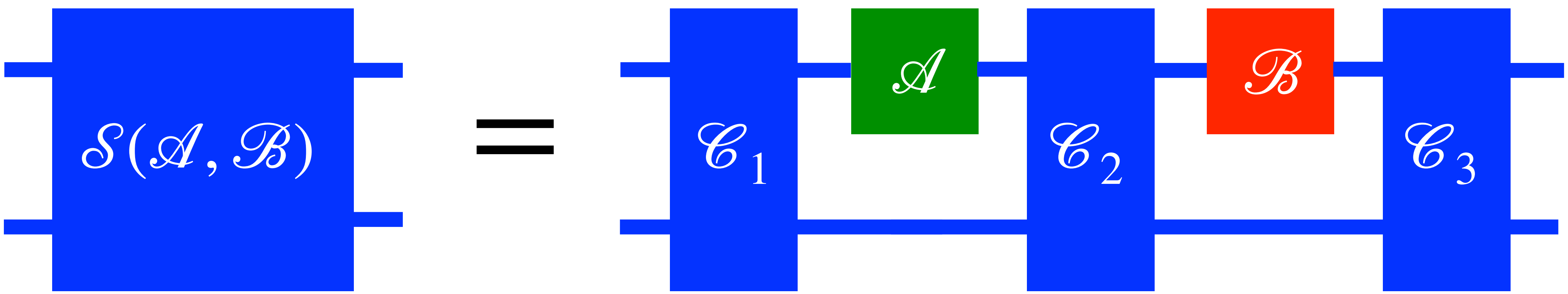
The quantum SWITCH produces a new channel $\mathscr{S}(\mathscr{A},\mathscr{B})$ with Kraus operators

$$S_{ij} := A_i B_j \otimes |0\rangle\langle 0| + B_j A_i \otimes |1\rangle\langle 1|$$

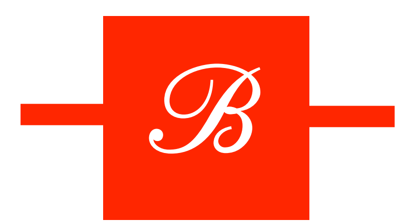
INCOMPATIBILITY WITH FIXED CAUSAL ORDER

Theorem (CDPV 2009/2013)

It is impossible to find quantum channels $\mathscr{C}_1$, $\mathscr{C}_2$, and $\mathscr{C}_3$ such that



for all unitary 

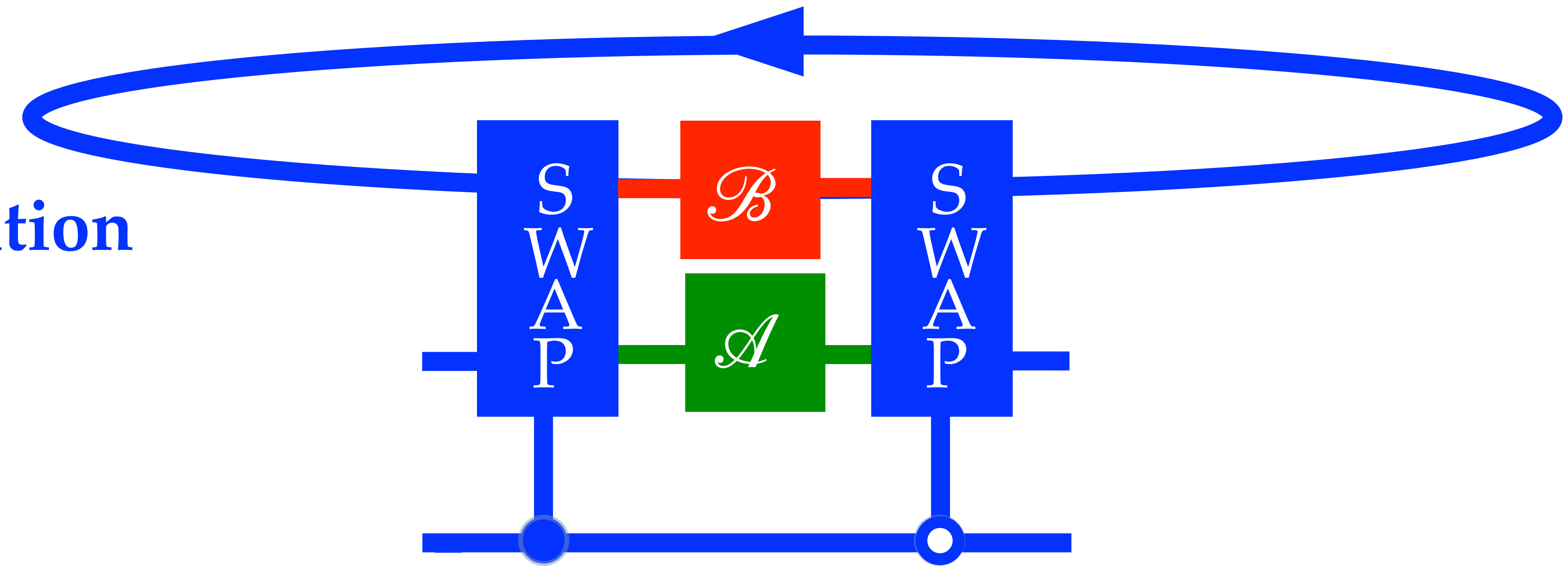
and 

(same holds with $\mathscr{A}$ and $\mathscr{B}$ in the opposite order, and for classical mixtures of the two orders)

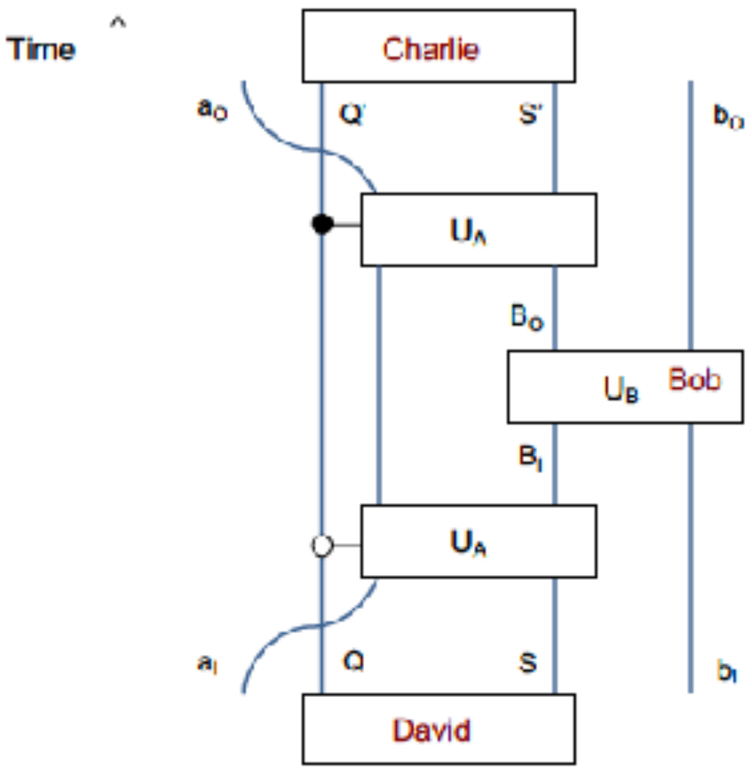
REALIZATIONS / SIMULATIONS

Time loops/
Postselected teleportation

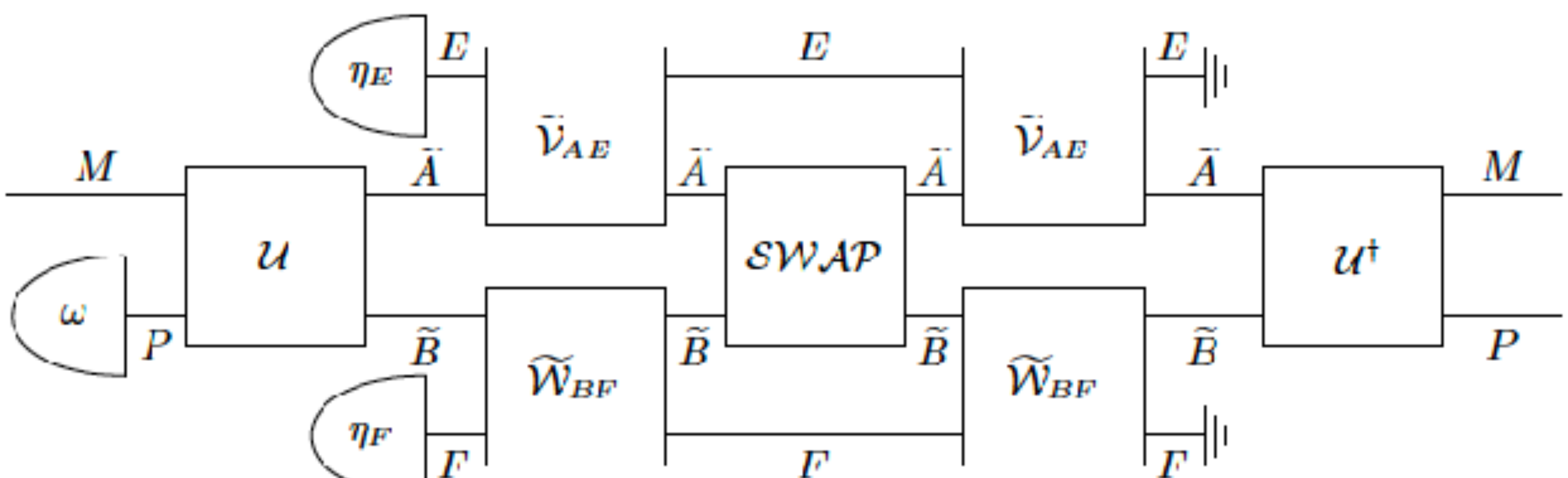
CDPV 2009/2013



Interferometric-type



With controlled operations
Oreshkov, Quantum 3, 206 (2019)



With modes and the vacuum
Chiribella and Kristjánsson, Proceedings of the Royal Society A, 475, 20180903 (2019)

APPLICATIONS
OF
THE QUANTUM SWITCH

REDUCING QUERY COMPLEXITY

- Testing commutativity

e.g. ***discover if operators commute or anti-commute***

probability of correct answer $= 1$ with the quantum SWITCH

< 1 for every testing strategy where $\mathscr{A}$ and $\mathscr{B}$ are connected in a definite order.

Chiribella, PRA 86, 040301(R) (2012)

Generalization to witnesses of indefinite causal order:

Araújo *et al*, NJP 17 10 (2015)

Extension to N channels:

Araujo, Costa, and Brukner, PRL 113, 250402 (2014)

QUANTUM COMMUNICATION ASSISTED BY THE QUANTUM SWITCH

CLASSICAL SHANNON THEORY

Claude E. Shannon

The carriers of information are classical:
classical states, classical channels,
classical configuration of the channels

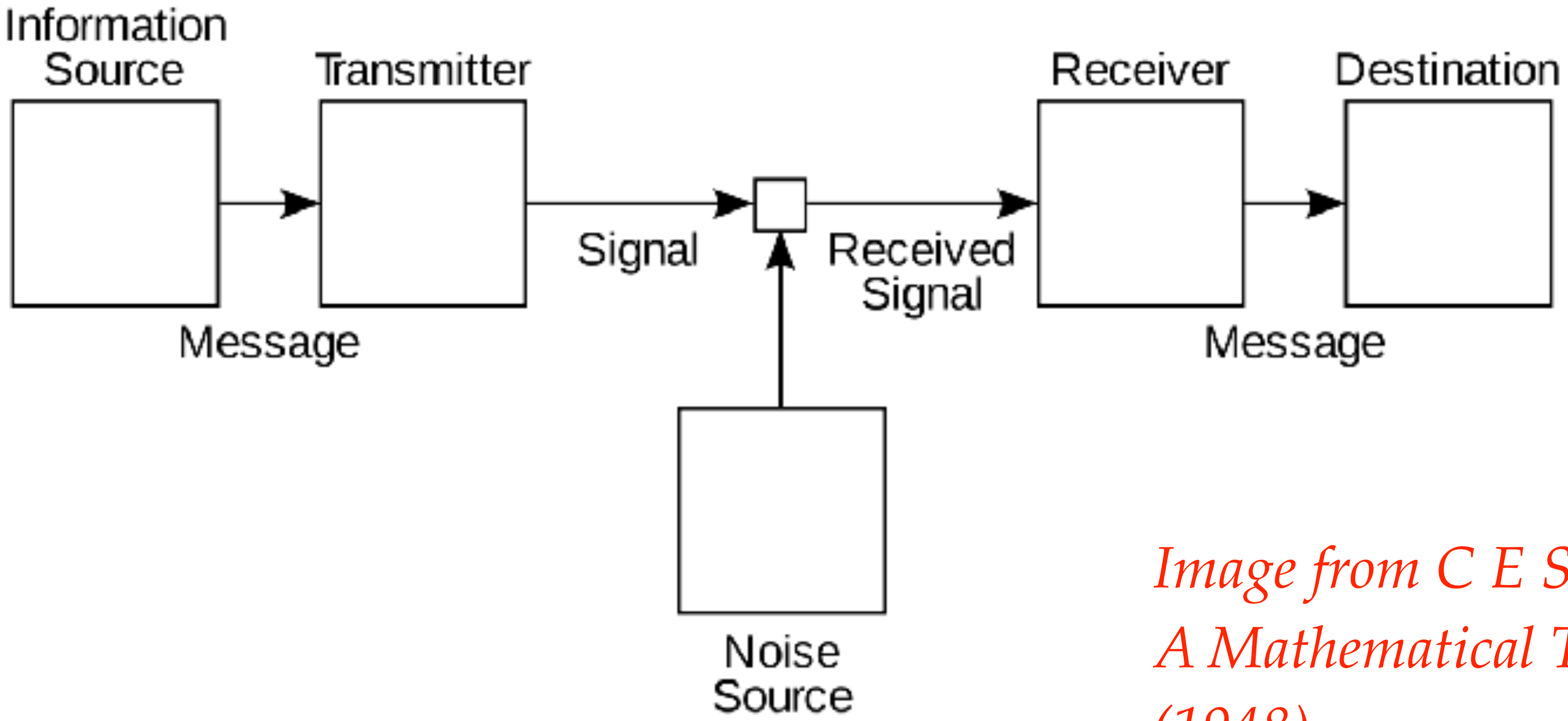


Image from C E Shannon,
A Mathematical Theory of Communication
(1948)

QUANTUM SHANNON THEORY

Allows the state of the information carriers and the channels be quantum.

Messages can be quantum:
not just strings of bits, like 0010110111,
but also quantum superpositions, like

$$|\Psi\rangle = \frac{|0010110111\rangle + |1010100011\rangle}{\sqrt{2}}$$

Still, the configuration of the communication channels is fixed.

Alexander Holevo

Benjamin Schumacher

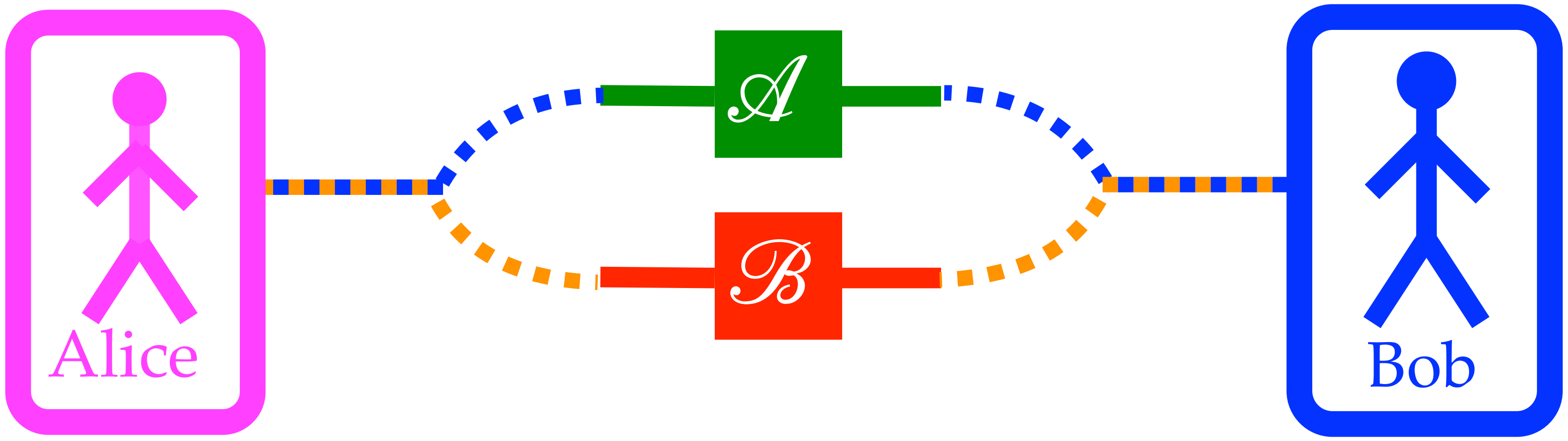
Charles Bennett

QUANTUM CONFIGURATIONS

Quantum theory allows

quantum-controlled configurations of the communication channels.

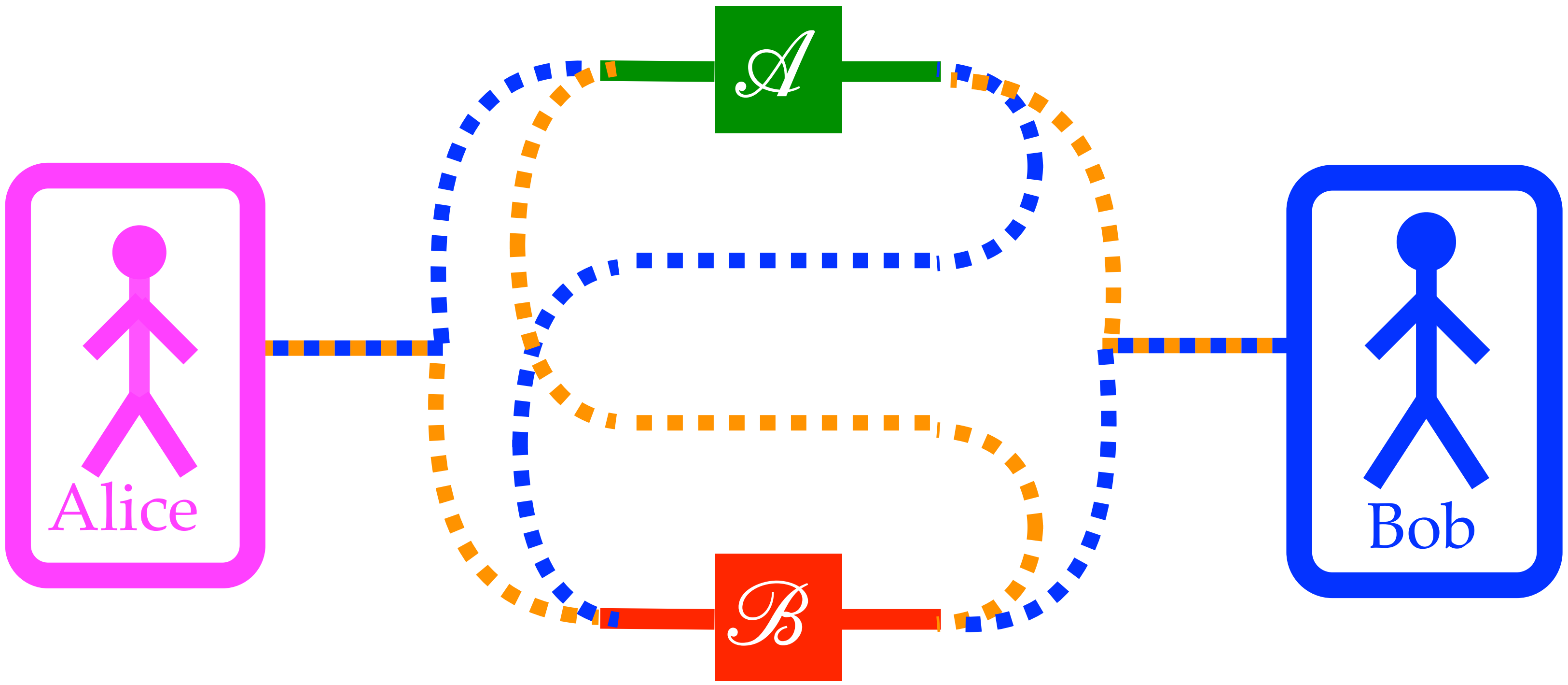
- **Example 1**: message in a superposition of going through one communication channel or another



Aharonov, Anandan, Popescu, Vaidman, PRL 64, 2965 (1990), **Oi**, PRL 91, 067902 (2003)
Gisin, Linden, Massar, Popescu PRA 72, 012338 (2005)
Abbott, Wechs, Horsman, Mhalla, Branciard, Quantum 4, 333 (2020)
Chiribella and Kristjánsson, Proc. Royal Soc. A 475, 20180903 (2019)
Dong, Nakayama, Soeda, Murao, arXiv:1911.01645 (2019)
Kristjánsson and Chiribella, Phys. Rev. Research 3, 043147 (2021)
…

QUANTUM CONFIGURATIONS

- **Example 2 (this talk)**:
  message traversing two channels in a superposition of alternative causal orders

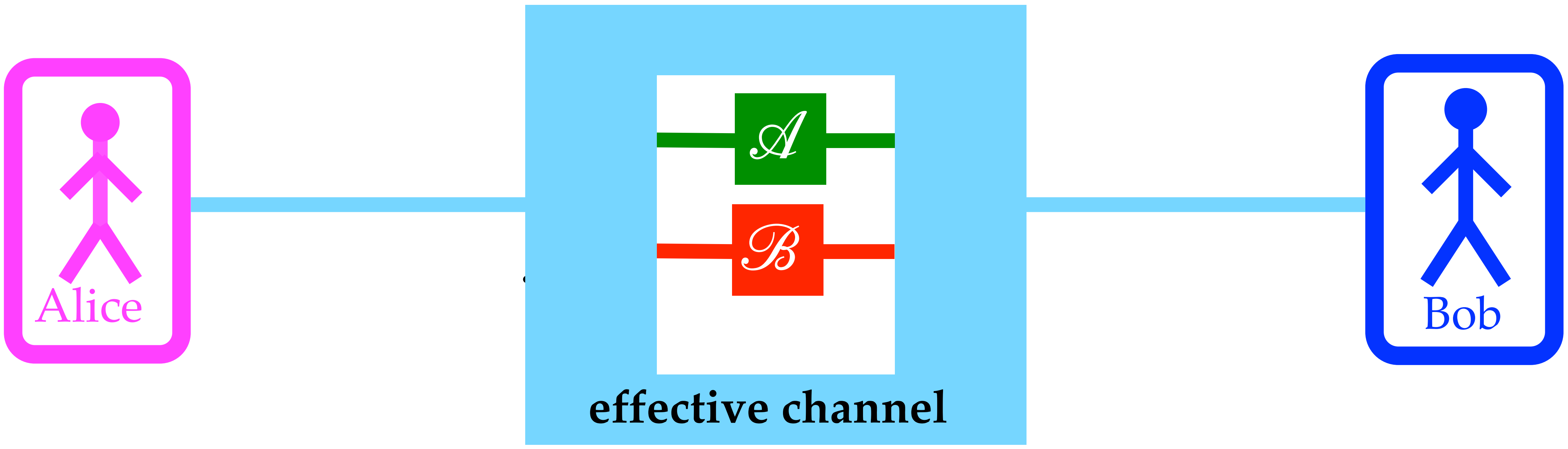


Ebler, Salek, Chiribella, Phys. Rev. Lett. 120, 120502 (2018)
Goswami et al, Phys. Rev. Res. 2, 033292 (2020).
Salek, Ebler, Chiribella, arXiv:1809.06655
Chiribella, Banik, Bhattacharya, Guha, Alimuddin, Roy, Saha, Agrawal, arXiv:1810.10457; New Journal of Physics 23, 033039 (2021).
N. Loizeau and A. Grinbaum, Physical Review A 101, 012340 (2020).
Caleffi and Cacciapuoti, IEEE J. Sel. Areas Commun. 38, 575 (2020).
Bhattacharya, Maity, Guha, Chiribella, Banik, PRX Quantum 2, 020350 (2021), …

GENERAL SETTINGS

- Some communication devices are available to a **communication provider.** Each device is described by a quantum channel.

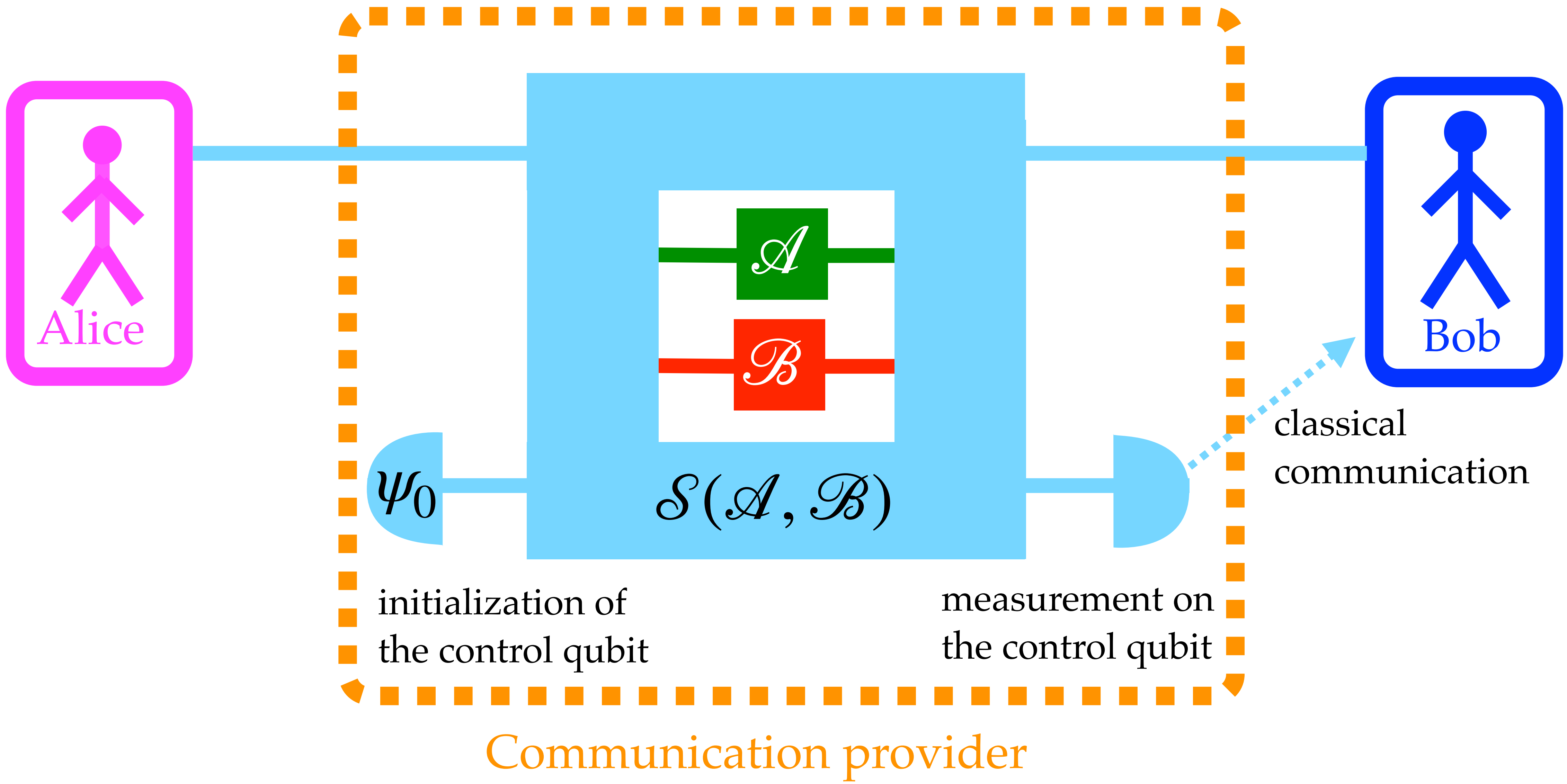
- The communication provider assembles the devices in a network, building an **effective channel** that connects a sender to a receiver.



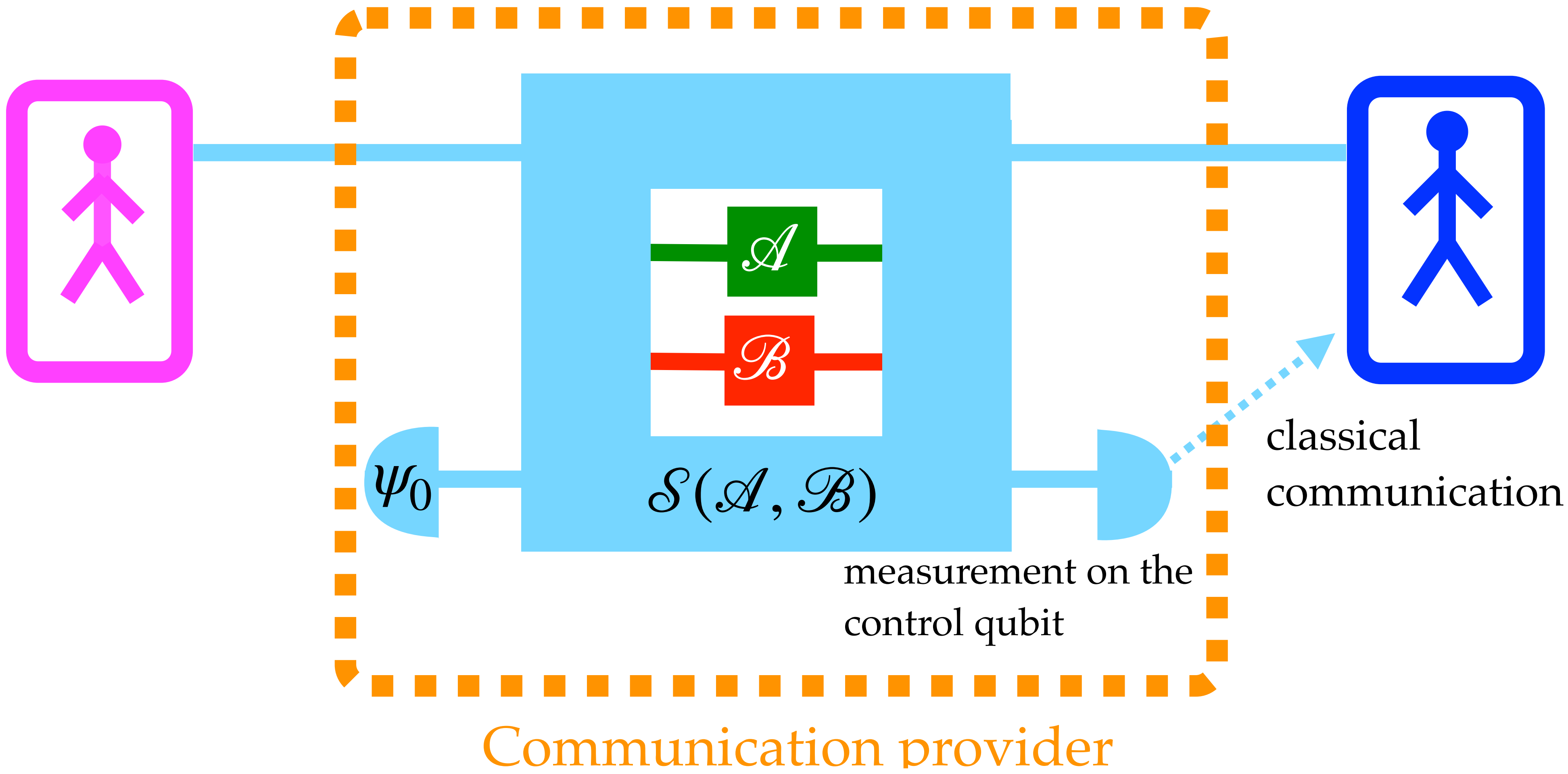
Kristjánsson, Chiribella, Salek, Ebler, and Wilson, New Journal of Physics 22, 073014 (2020).

COMMUNICATION WITH QUANTUM SWITCH

Communication provider combines two devices in the quantum SWITCH, and uses the control system to assist the communication between a sender and a receiver.



Chiribella, Banik, Bhattacharya, Guha, Alimuddin, Roy, Saha, Agrawal, Kar,
arXiv:1810.10457, New Journal of Physics 23, 033039 (2021).



This setting is similar to the setting of
quantum communication with classical feedback from the environment.
Gregoratti and Werner, J. Mod. Opt. 50 915–33 (2003)

Important difference: *here we do not assume that the whole environment is accessible.*
The only accessible part of the environment is a two-dimensional system responsible for the order of channels $\mathscr{A}$ and $\mathscr{B}$.
The environments of $\mathscr{A}$ and $\mathscr{B}$ are inaccessible.

EXAMPLE: COMPLETELY DEPOLARIZING CHANNELS

Ebler, Salek, Chiribella, Phys. Rev. Lett. 120, 120502 (2018)

COMPLETELY DEPOLARIZING CHANNELS

Suppose we are given
two completely depolarizing channels (CDCs)

$$\mathscr{A} = \mathscr{B} \quad \text{with} \quad \mathscr{A}(\rho) = \mathscr{B}(\rho) = \frac{I}{d} \quad \forall \rho$$

In standard quantum Shannon theory,
these two channels are useless.

Their **classical capacity is 0 bits.**

TWO CDCS IN A QUANTUM SWITCH

Input channels: $\mathscr{A}(\rho) = \mathscr{B}(\rho) = \dfrac{I}{d}$

Initial state of control qubit: $|\psi_0\rangle = \sqrt{p}\,|0\rangle + \sqrt{1-p}\,|1\rangle$

Effective channel from Alice to Bob + Charlie

$$\mathscr{C}_{\text{eff}}(\rho) = p\,\frac{I}{d} \otimes |0\rangle\langle 0| + (1-p)\frac{I}{d} \otimes |1\rangle\langle 1|$$

$$\boxed{+\sqrt{p(1-p)}\,\frac{\rho}{d^2} \otimes |0\rangle\langle 1| + \text{h.c.}}$$

interference term

CLASSICAL CAPACITY

A **lower bound** to the classical capacity is the Holevo information

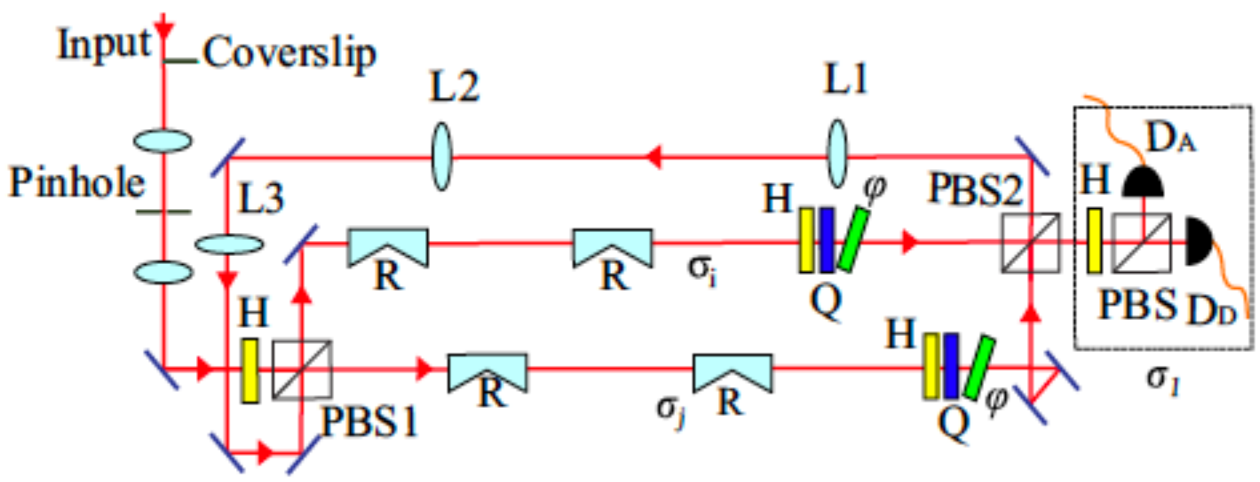
$$\chi(\mathscr{C}_{\text{eff}}) = \log d - p\log p - (1-p)\log(1-p)$$

$$+\left\{\left(\frac{d+1}{2d^2}\right)\log\left(\frac{d+1}{2d^2}\right)+\left(\frac{d-1}{2d^2}\right)\log\left(\frac{d-1}{2d^2}\right)+2(d-1)\left(\frac{1}{2d}\right)\log\left(\frac{1}{2d}\right)\right\}$$

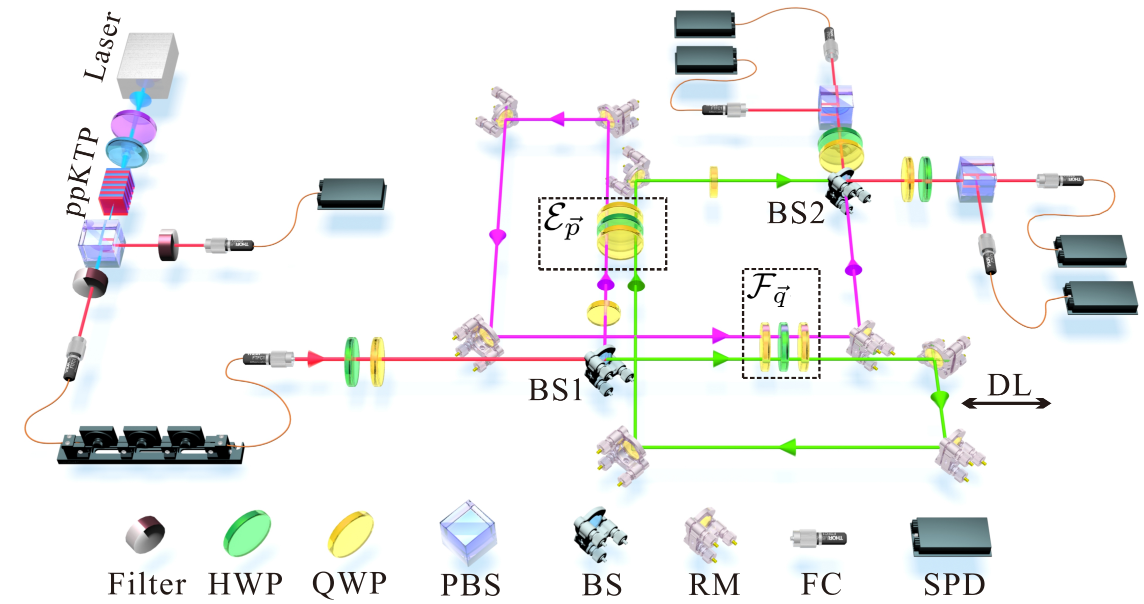
- **it is non-zero (for qubits 0.049 bits)**
- **it is maximum for $p = 1/2$ (uniform superposition of orders)**
- **it decreases with d**
- **later proven to be equal to the capacity**

Chiribella, Wilson, and Chau, Phys. Rev. Lett. 127, 190502 (2021)

EXPERIMENTS



Polarization control, spatial modes target
Goswami, Cao, Paz-Silva, Romero, and White
Phys. Rev. Research 2, 033292 (2020)



Path control, polarization target
Guo et al, Phys. Rev. Lett. 124, 030502 (2020).

Review:

Experiments on quantum causality

AVS Quantum Sci. 2, 037101 (2020): https://doi.org/10.1116/5.0010747

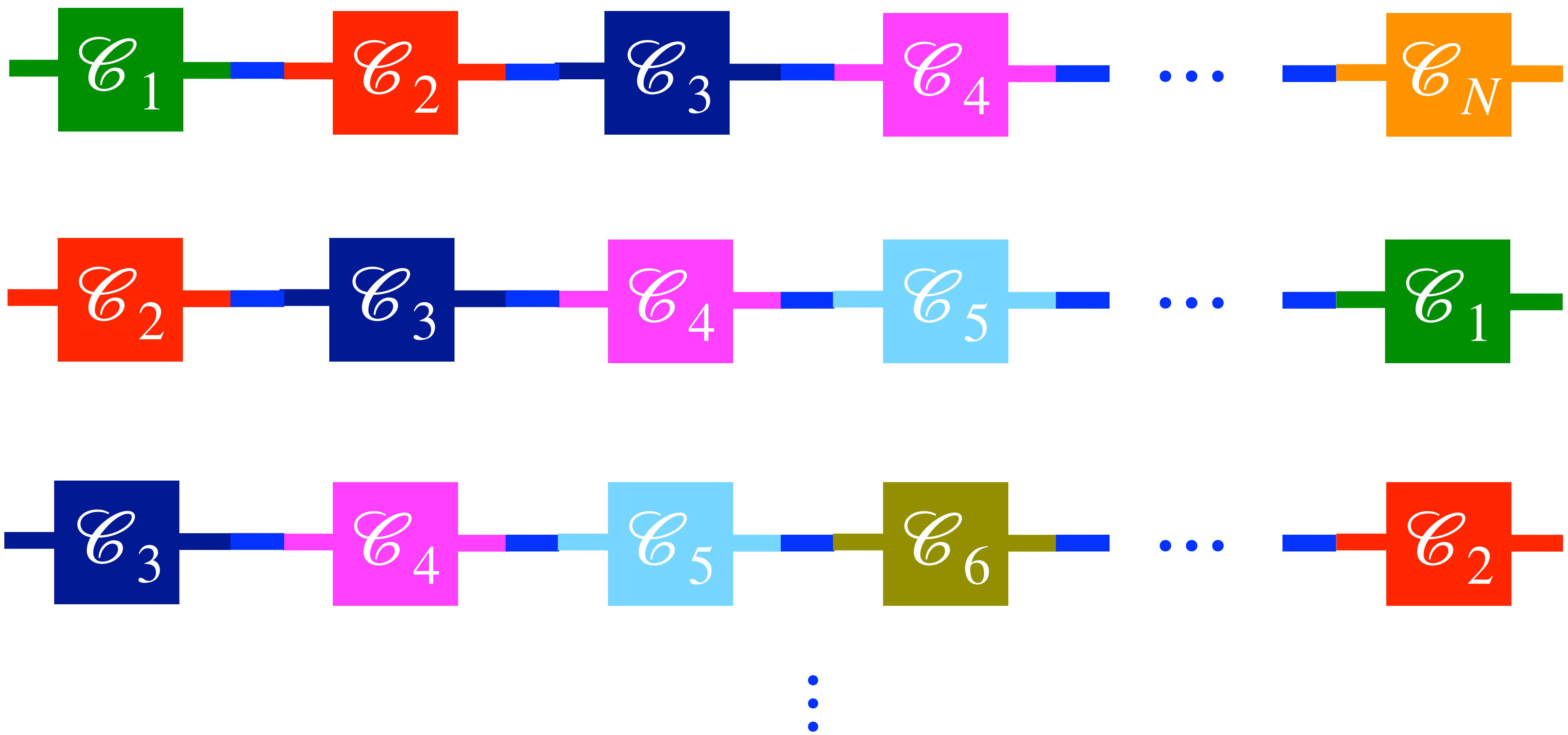
K. Goswami[a] and J. Romero[b]

QUANTUM COMMUNICATION THROUGH *N* CDCS IN *N* CYCLIC ORDERS

Chiribella, Wilson, and Chau, Phys. Rev. Lett. 127, 190502 (2021)

CYCLIC PERMUTATIONS

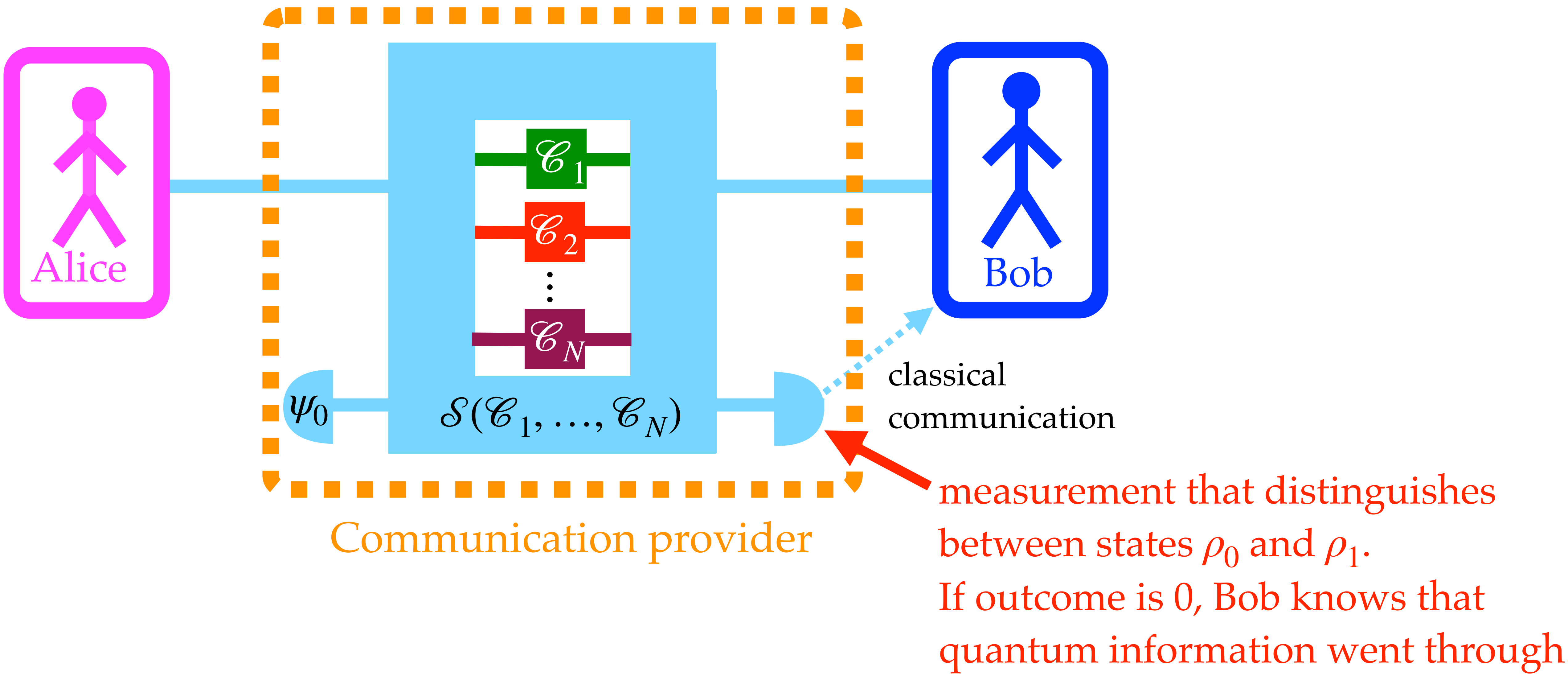
For N≥2 noisy channels, consider communication protocols with quantum control on the N cyclic permutations



L. M. Procopio, F. Delgado, M. Enríquez, N. Belabas, and J. A. Levenson, Physical Review A 101, 012346 (2020).
S. Sazim, M. Sedlak, K. Singh, and A. K. Pati, Physical Review A 103, 062610 (2021).
M. Wilson and G. Chiribella, Electronic Proceedings in Theoretical Computer Science 340, 333 (2021).

HERALDED TRANSMISSION OF QUANTUM STATES

For large N, **quantum states can be transmitted** with success probability $\dfrac{1}{d^2} + O(1/N)$ and error vanishing as $O(d^2/N)$



FINITE *N* REGIME

Quantum information can be transmitted reliably
if N is **sufficiently large.**

How large?

- if $N \leq d + 1$, quantum information **cannot be transmitted.**

- if $N > d + 1$, quantum information **can be transmitted with the assistance of two-way classical communication.**

cf. Bennett, DiVincenzo, Smolin, and Wootters, Physical Review A 54, 3824 (1996).

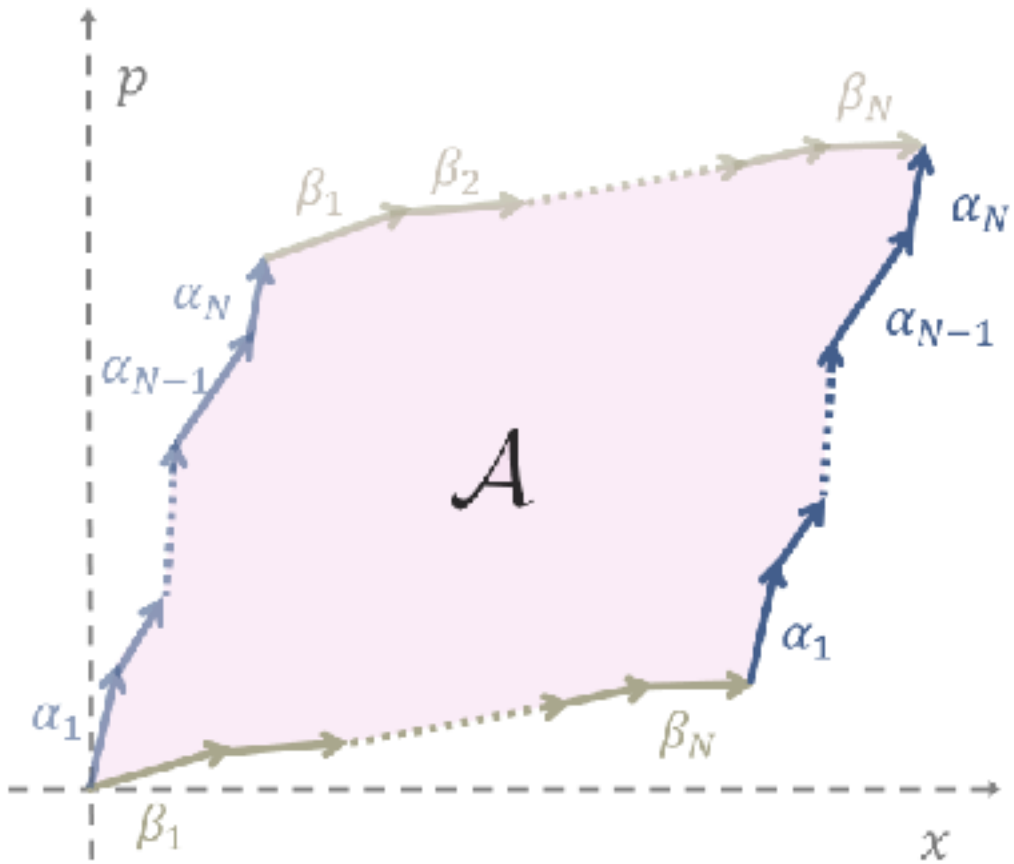
For qubits, N= 4 is enough!

Potential experiment with the techniques of Taddei et al., PRX Quantum 2, 010320 (2021).

QUANTUM METROLOGY WITH INDEFINITE CAUSAL ORDER

Zhao, Yang, and Chiribella, Phys. Rev. Lett. 124, 190503 (2020)

ESTIMATION OF A PHASE SPACE AREA



Settings: A harmonic oscillator is subject to two sets of phase space displacements.
The task is to estimate the phase space area enclosed by the corresponding paths.

For concreteness, consider position and momentum displacements

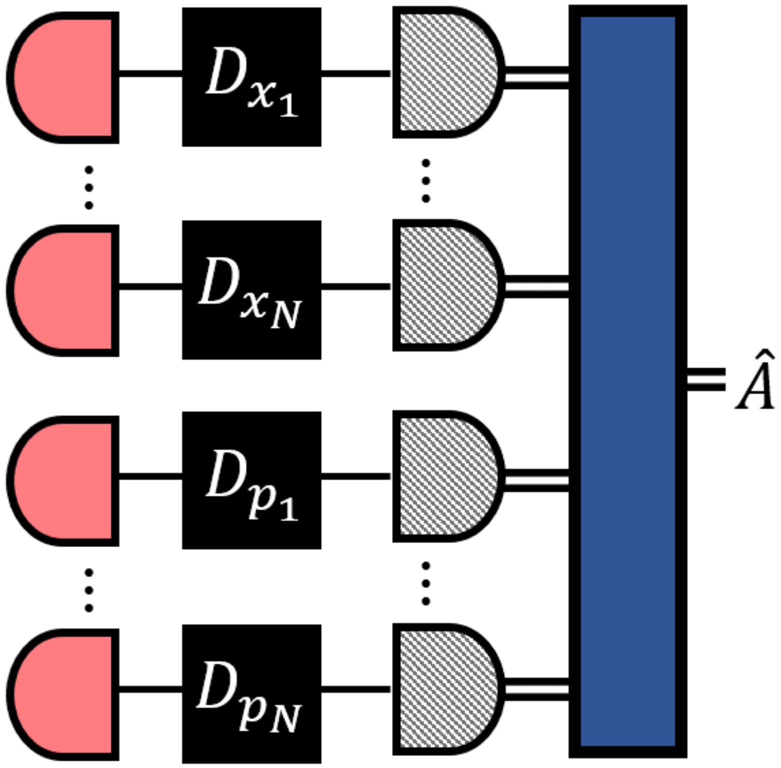
$$D_{x_j} = \exp[-ix_j P] \qquad j \in \{1, \ldots, N\}$$

$$D_{p_k} = \exp[ip_k X] \qquad k \in \{1, \ldots, N\}$$

Task: estimate the regularized area $A := \mathscr{A}/N^2$

CAUSALLY ORDERED STRATEGIES (1)

Strategy 1: estimate each displacement independently



For a single displacement *z*,
the *Root Mean Square Error (RMSE)* is

$$\Delta z = \frac{1}{\sqrt{8\nu E}} \quad \text{with}$$

ν = number of repetitions of the experiment

$E = \langle X^2 + P^2 \rangle / 2$ = average energy of the probe

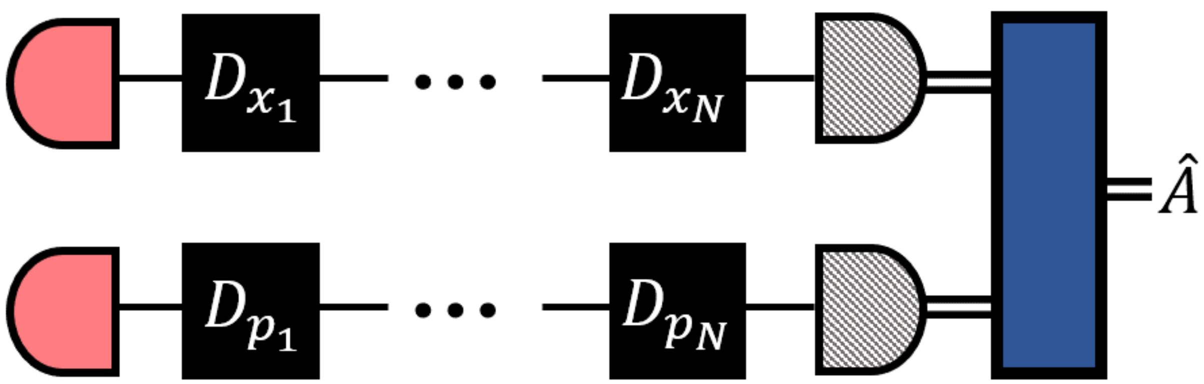
The averages and their product are computed classically.

RMSE for the product: $\Delta A = O\left(\frac{1}{\sqrt{\nu N}}\right)$

(standard quantum limit w.r.t. *N*)

CAUSALLY ORDERED STRATEGIES (2)

Strategy 2: **separately estimate the total displacements in x and p**



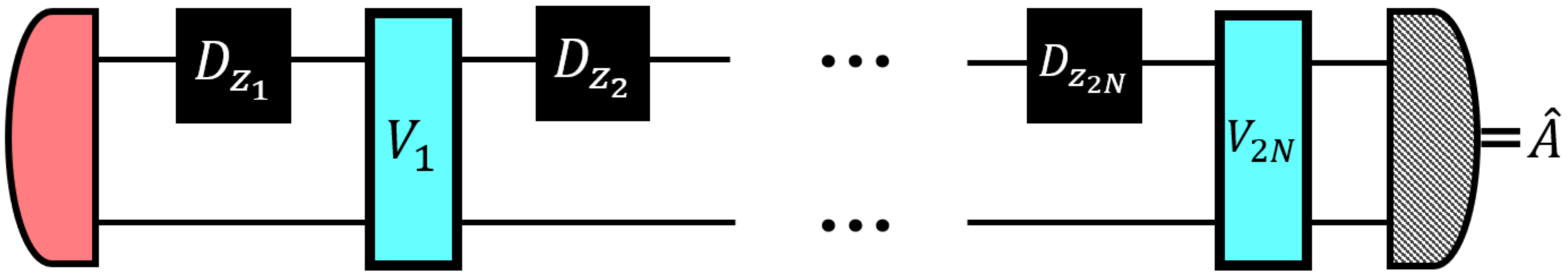
The product is then computed classically.

RMSE for the product: $\Delta A = O\left(\dfrac{1}{\sqrt{\nu}N}\right)$

(Heisenberg limit w.r.t. N)

CAUSALLY ORDERED STRATEGIES (3)

Most general causally ordered strategy



General bound on RMSE: $$\Delta A_{\text{fixed}} = \Omega\left(\frac{1}{\sqrt{\nu}N}\right)$$

No causally ordered scheme (with bounded energy) can beat Heisenberg limit w.r.t. N

ADVANTAGE OF THE QUANTUM SWITCH

For small A, the quantum SWITCH yields

$$\Delta A_{\text{switch}} = \frac{1}{\sqrt{\nu} N^2}$$

to be contrasted with the bound $\Delta A_{\text{fixed}} = \Omega\left(\frac{1}{\sqrt{\nu} N}\right)$ for general strategies with definite causal order.

In general, the quantum SWITCH enables estimation of the phase

$$\phi = \sum_{i,j} x_i p_j \mod 2\pi \quad \text{with error} \quad \Delta\phi = \frac{1}{\sqrt{\nu}}$$

whereas causally-ordered strategies have error $\Delta\phi = \Omega\left(\frac{N}{\sqrt{\nu}}\right)$

IN SHORT

entanglement $\longrightarrow$ quadratic speedup over unentangled strategies (Heisenberg limit)

indefinite causal order $\longrightarrow$ further quadratic speedup over causally ordered strategies (super-Heisenberg scaling)

EXPERIMENT

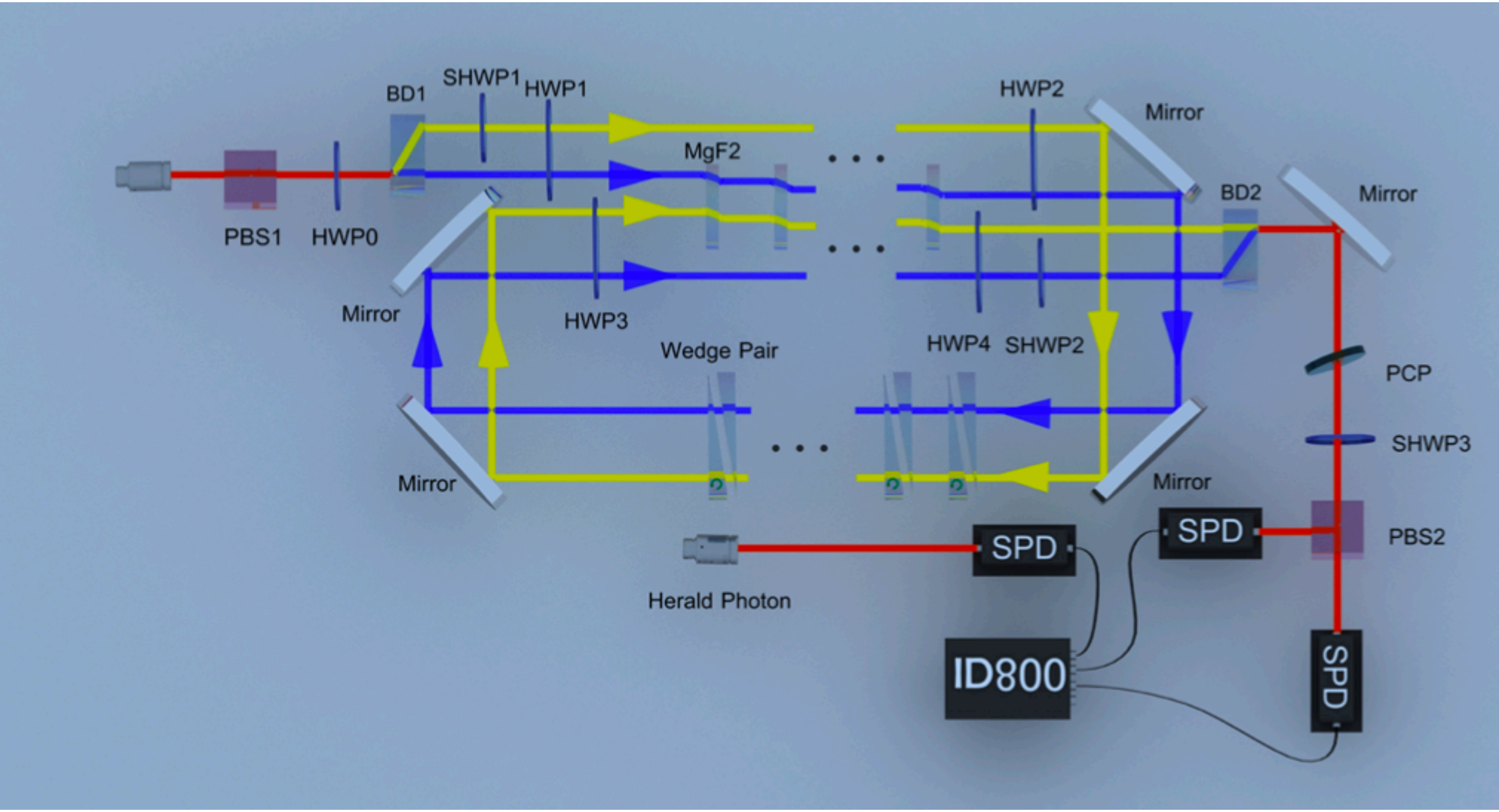


Image from Yin et al, Experimental super-Heisenberg quantum metrology with indefinite gate order, https://www.researchsquare.com/article/rs-1327792/v1

FUTURE DIRECTIONS

OUTLOOK

- Quantum communication
  -multiparty protocols
  -transition to quantum thermodynamics

  Felce and Vedral, Phys. Rev. Lett. 125, 070603 (2020)
  Guha, Alimuddin, and Parashar, Phys. Rev. A 102, 032215 (2020)
  Simonov, Francica, Guarnieri, and Paternostro, Phys. Rev. A 105, 032217 (2022)

- Quantum metrology
  -super-Heisenberg scaling with qubits?
  -more robustness in noisy metrology?
  -more practical applications?